

Ohio's State Tests

ITEM RELEASE

SPRING 2017

GEOMETRY

Table of Contents

Questions 1 – 21: Content Summary and Answer Key	iii
Question 1: Question and Scoring Guidelines.....	1
Question 1: Sample Response	4
Question 2: Question and Scoring Guidelines.....	5
Question 2: Sample Responses	7
Question 3: Question and Scoring Guidelines	15
Question 3: Sample Response	18
Question 4: Question and Scoring Guidelines.....	19
Question 4: Sample Responses	23
Question 5: Question and Scoring Guidelines.....	35
Question 5: Sample Responses	37
Question 6: Question and Scoring Guidelines.....	41
Question 6: Sample Responses	45
Question 7: Question and Scoring Guidelines.....	53
Question 7: Sample Responses	57
Question 8: Question and Scoring Guidelines.....	63
Question 8: Sample Response	65
Question 9: Question and Scoring Guidelines.....	67
Question 9: Sample Response	69
Question 10: Question and Scoring Guidelines.....	71
Question 10: Sample Responses	75
Question 11: Question and Scoring Guidelines.....	81
Question 11: Sample Response	83
Question 12: Question and Scoring Guidelines.....	85
Question 12: Sample Response	88
Question 13: Question and Scoring Guidelines.....	89
Question 13: Sample Responses	93

Question 14: Question and Scoring Guidelines.....	101
Question 14: Sample Response	103
Question 15: Question and Scoring Guidelines.....	105
Question 15: Sample Responses	107
Question 16: Question and Scoring Guidelines.....	113
Question 16: Sample Response	115
Question 17: Question and Scoring Guidelines.....	117
Question 17: Sample Response	120
Question 18: Question and Scoring Guidelines.....	121
Question 18: Sample Response	124
Question 19: Question and Scoring Guidelines.....	125
Question 19: Sample Responses	127
Question 20: Question and Scoring Guidelines.....	135
Question 20: Sample Responses	139
Question 21: Question and Scoring Guidelines.....	145
Question 21: Sample Responses	149

Geometry
Spring 2017 Item Release
Content Summary and Answer Key

Question No.	Item Type	Content Cluster	Content Standard	Answer Key	Points
1	Multiple Choice	Visualize relationships between two-dimensional and three-dimensional objects.	Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects. (G.GMD.4)	A	1 point
2	Equation Item	Use coordinates to prove simple geometric theorems algebraically and to verify specific geometric statements.	Find the point on a directed line segment between two given points that partitions the segment in a given ratio. (G.GPE.6)	---	1 point
3	Multiple Choice	Prove and apply theorems both formally and informally involving similarity using a variety of methods.	Use congruence and similarity criteria for triangles to solve problems and to justify relationships in geometric figures that can be decomposed into triangles. (G.SRT.5)	A	1 point

Geometry
Spring 2017 Item Release
Content Summary and Answer Key

Question No.	Item Type	Content Cluster	Content Standard	Answer Key	Points
4	Short Response	Understand independence and conditional probability, and use them to interpret data.	Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. <i>For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.</i> (S.CP.5)	---	2 points
5	Equation Item	Explain volume formulas, and use them to solve problems.	Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems. (G.GMD.3)	---	1 point
6	Equation Item	Understand similarity in terms of similarity transformations.	Verify experimentally the properties of dilations given by a center and a scale factor. (G.SRT.1) b. The dilation of a line segment is longer or shorter in the ratio given by the scale factor.	---	1 point
7	Equation Item	Apply geometric concepts in modeling situations.	Apply concepts of density based on area and volume in modeling situations, e.g., persons per square mile, BTUs per cubic foot. (G.MG.2)	---	1 point

Geometry
Spring 2017 Item Release
Content Summary and Answer Key

Question No.	Item Type	Content Cluster	Content Standard	Answer Key	Points
8	Multiple Choice	Understand congruence in terms of rigid motions.	Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent. (G.CO.6)	D	1 point
9	Multiple Choice	Use the rules of probability to compute probabilities of compound events in a uniform probability model.	Find the conditional probability of A given B as the fraction of B's outcomes that also belong to A, and interpret the answer in terms of the model. (S.CP.6)	A	1 point
10	Equation Item	Understand and apply theorems about circles.	Construct the inscribed and circumscribed circles of a triangle; prove and apply the property that opposite angles are supplementary for a quadrilateral inscribed in a circle. (G.C.3)	---	1 point
11	Multiple Choice	Understand independence and conditional probability, and use them to interpret data.	Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not"). (S.CP.1)	A	1 point

Geometry
Spring 2017 Item Release
Content Summary and Answer Key

Question No.	Item Type	Content Cluster	Content Standard	Answer Key	Points
12	Multi-Select Item	Experiment with transformation in the plane.	Identify the symmetries of a figure, which are the rotations and reflections that carry it onto itself. (G.CO.3)	B, C, E	1 point
13	Hot Text Item	Prove geometric theorems both formally and informally using a variety of methods.	Prove and apply theorems about parallelograms. <i>Theorems include but are not restricted to the following: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.</i> (G.CO.11)	---	1 point
14	Multiple Choice	Define trigonometric ratios, and solve problems involving right triangles.	Explain and use the relationship between the sine and cosine of complementary angles. (G.SRT.7)	D	1 point
15	Equation Item	Use coordinates to prove simple geometric theorems algebraically and to verify specific geometric statements.	Justify the slope criteria for parallel and perpendicular lines, and use them to solve geometric problems, e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point. (G.GPE.5)	---	1 point

Geometry
Spring 2017 Item Release
Content Summary and Answer Key

Question No.	Item Type	Content Cluster	Content Standard	Answer Key	Points
16	Multiple Choice	Experiment with transformations in the plane.	Know precise definitions of ray, angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and arc length. (G.CO.1)	B	1 point
17	Multiple Choice	Prove and apply theorems both formally and informally involving similarity using a variety of methods.	Prove and apply theorems about triangles. <i>Theorems include but are not restricted to the following: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.</i> (G.SRT.4)	B	1 point
18	Multi-Select Item	Understand congruence in terms of rigid motions.	Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent. (G.CO.7)	A, C, D, E	1 point
19	Equation Item	Translate between the geometric description and the equation for a conic section.	Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation. (G.GPE.1)	---	1 point

Geometry
Spring 2017 Item Release
Content Summary and Answer Key

Question No.	Item Type	Content Cluster	Content Standard	Answer Key	Points
20	Graphic Response Item	Experiment with transformation in the plane.	Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using items such as graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another. (G.CO.5)	---	1 point
21	Equation Item	Define trigonometric ratios, and solve problems involving right triangles.	Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles. (G.SRT.6)	---	1 point

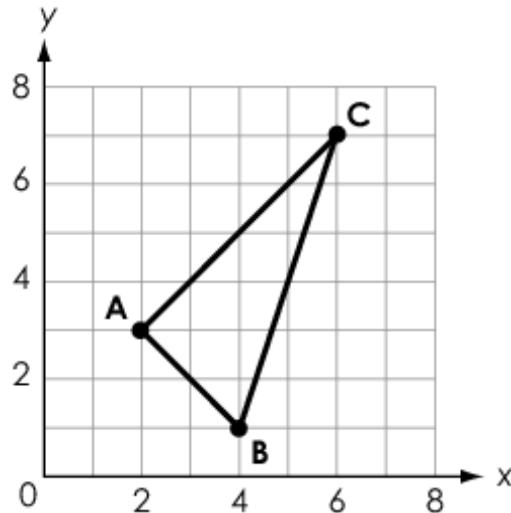
Geometry
Spring 2017 Item Release

Question 1

Question and Scoring Guidelines

Question 1

Triangle ABC is shown.



Which three-dimensional figure results from rotating the triangle 360° about $\overline{AC}$?

- Ⓐ cone
- Ⓑ cylinder
- Ⓒ pyramid
- Ⓓ sphere

Points Possible: 1

Content Cluster: Visualize relationships between two-dimensional and three-dimensional objects.

Content Standard: Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.
(G.GMD.4)

Scoring Guidelines

Rationale for Option A: **Key** – The student correctly identified that when rotating the triangle 360 degrees about side AC, side AB would result in a flat circular base, and that the lateral face would be triangular in nature with a curved face that ends in an apex point, which is a three-dimensional figure called a cone.

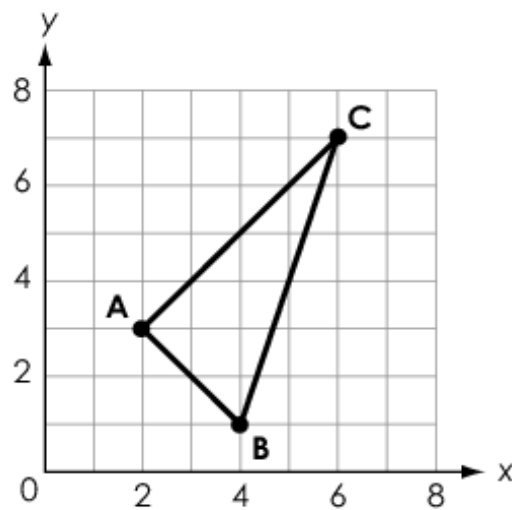
Rationale for Option B: This is incorrect. The student may have thought that both bases of the object would be a circle and has not realized that a curved face would end in an apex point.

Rationale for Option C: This is incorrect. The student may have thought that when rotating the triangle 360 degrees about side AC, side AB would result in a flat polygonal base instead of a flat circular base, and thought that the resulting three-dimensional solid was a pyramid.

Rationale for Option D: This is incorrect. The student identified that when rotating the triangle 360 degrees about side AC, side AB would result in a flat circular base, but then extended that reasoning to all directions instead of limiting it to just the base.

Sample Response: 1 point

Triangle ABC is shown.



Which three-dimensional figure results from rotating the triangle 360° about $\overline{AC}$?

- ☒ cone
- ☐ (B) cylinder
- ☐ (C) pyramid
- ☐ (D) sphere

Geometry
Spring 2017 Item Release

Question 2

Question and Scoring Guidelines

Question 2

Line segment AB has endpoints A $(-1.5, 0)$ and B $(4.5, 8)$. Point C is on line segment AB and is located at $(0, 2)$.

What is the ratio of $\frac{AC}{CB}$?

← → ↶ ↷ ✕

1	2	3
4	5	6
7	8	9
	0	
.	-	$\frac{\Box}{\Box}$

Points Possible: 1

Content Cluster: Use coordinates to prove simple geometric theorems algebraically and to verify specific geometric statements.

Content Standard: Find the point on a directed line segment between two given points that partitions the segment in a given ratio. (G.GPE.6)

Scoring Guidelines

Exemplar Response

- $\frac{1}{3}$

Other Correct Responses

- Any equivalent ratio
- Any decimal value 0.33, 0.333, 0.3333, etc.

For this item, a full-credit response includes:

- The correct ratio (1 point).

Geometry
Spring 2017 Item Release

Question 2

Sample Responses

Sample Response: 1 point

Line segment AB has endpoints A $(-1.5, 0)$ and B $(4.5, 8)$. Point C is on line segment AB and is located at $(0, 2)$.

What is the ratio of $\frac{AC}{CB}$?

$\frac{1}{3}$



1	2	3
4	5	6
7	8	9
	0	
.	-	$\frac{\Box}{\Box}$

Notes on Scoring

This response earns full credit (1 point) because it shows a correct ratio of lengths of $\overline{AC}$ and $\overline{CB}$, $\frac{AC}{CB}$, or $\frac{1}{3}$.

There are different ways to find the ratio. One of them is to find the lengths of $\overline{AC}$ and $\overline{CB}$ using the distance formula,

$$\begin{aligned} AC &= \sqrt{((-1.5 - 0)^2 + (0 - 2)^2)} \\ &= \sqrt{(-1.5)^2 + (-2)^2} \\ &= \sqrt{6.25} \\ &= 2.5 \text{ and} \end{aligned}$$

$$\begin{aligned} CB &= \sqrt{((0 - 4.5)^2 + (2 - 8)^2)} \\ &= \sqrt{(-4.5)^2 + (-6)^2} \\ &= \sqrt{56.25} \\ &= 7.5. \end{aligned}$$

The ratio of $\frac{AC}{CB}$ is $\frac{2.5}{7.5}$ or $\frac{1}{3}$.

Another way is based on the notion that the ratio of $\frac{AC}{CB}$ is equal to the ratio of the corresponding lengths of the line segments along the x-axis: $\frac{(0 - (-1.5))}{(4.5 - 0)}$ or $\frac{1.5}{4.5}$ or $\frac{1}{3}$. The same would be true about the corresponding lengths of the line segments along the y-axis:

$$\begin{aligned} &\frac{(2 - 0)}{(8 - 2)} \\ &= \frac{2}{6} \\ &= \frac{1}{3}. \end{aligned}$$

The ratio of $\frac{1}{3}$ expressed in form of a decimal value as 0.33, 0.333 and etc. are accepted.

Sample Response: 1 point

Line segment AB has endpoints A $(-1.5, 0)$ and B $(4.5, 8)$. Point C is on line segment AB and is located at $(0, 2)$.

What is the ratio of $\frac{AC}{CB}$?

$$\frac{2.5}{7.5}$$



1	2	3
4	5	6
7	8	9
	0	
.	-	$\frac{\Box}{\Box}$

Notes on Scoring

This response earns full credit (1 point) because it shows a correct ratio of lengths of $\overline{AC}$ and $\overline{AB}$, $\frac{AC}{CB}$, or $\frac{1}{3}$.

There are different ways to find the ratio. One of them is to find the lengths of $\overline{AC}$ and $\overline{CB}$ using the distance formula,

$$AC = \sqrt{((-1.5 - 0)^2 + (0 - 2)^2)}$$

$$= \sqrt{(-1.5)^2 + (-2)^2}$$

$$= \sqrt{6.25}$$

$$= 2.5 \text{ and}$$

$$CB = \sqrt{((0 - 4.5)^2 + (2 - 8)^2)}$$

$$= \sqrt{(-4.5)^2 + (-6)^2}$$

$$= \sqrt{56.25}$$

$$= 7.5.$$

The ratio of $\frac{AC}{CB}$ is $\frac{2.5}{7.5}$.

Sample Response: 0 points

Line segment AB has endpoints A $(-1.5, 0)$ and B $(4.5, 8)$. Point C is on line segment AB and is located at $(0, 2)$.

What is the ratio of $\frac{AC}{CB}$?

10



1	2	3
4	5	6
7	8	9
	0	
.	-	$\frac{\Box}{\Box}$

Notes on Scoring

This response earns no credit (0 points) because it shows an incorrect ratio of $\frac{AC}{CB}$ as 10.

The student may have found the length of the line segment AB using the distance formula instead of finding the ratio of AC to CB,

$$AB = \sqrt{((4.5 + 1.5)^2 + (8 - 0)^2)}$$

$$= \sqrt{(6)^2 + (8)^2}$$

$$= \sqrt{100}$$

$$= 10.$$

Sample Response: 0 points

Line segment AB has endpoints A $(-1.5, 0)$ and B $(4.5, 8)$. Point C is on line segment AB and is located at $(0, 2)$.

What is the ratio of $\frac{AC}{CB}$?

0.148134



1	2	3
4	5	6
7	8	9
	0	
.	-	$\frac{\Box}{\Box}$

Notes on Scoring

This response earns no credit (0 points) because it shows an incorrect ratio of $\frac{AC}{CB}$ or 0.148134.

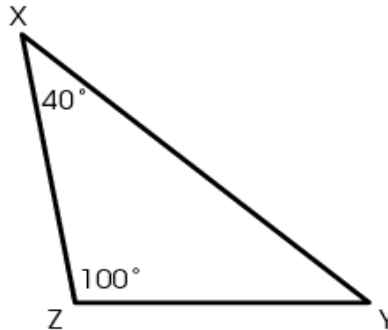
Geometry
Spring 2017 Item Release

Question 3

Question and Scoring Guidelines

Question 3

Triangle XYZ is shown.



Which triangle must be similar to triangle XYZ?

- (A) a triangle with two angles that measure 40°
- (B) a triangle with angles that measure 40° and 60°
- (C) a scalene triangle with only one angle that measures 100°
- (D) an isosceles triangle with only one angle that measures 40°

Points Possible: 1

Content Cluster: Prove and apply theorems both formally and informally involving similarity using a variety of methods.

Content Standard: Use congruence and similarity criteria for triangles to solve problems and to justify relationships in geometric figures that can be decomposed into triangles. (G.SRT.5)

Scoring Guidelines

Rationale for Option A: **Key** – The student correctly realized that the third angle of the given triangle is 40 degrees and applied the AA criterion to identify a similar triangle since the sum of the angles in any triangle is 180° .

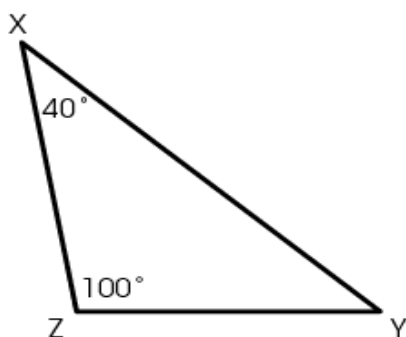
Rationale for Option B: This is incorrect. The student may have incorrectly found the measure of angle Y by subtracting 40° from 100° to get 60° ($100 - (40) = 60$), instead of subtracting the sum of 40° and 100° from 180° ($180 - (100+40)=40$), and then applied the AA criterion. The correct angle measure of Y is 40° instead of 60° .

Rationale for Option C: This is incorrect. The student may have made an assumption that triangle XYZ looks scalene and then incorrectly concluded that any two scalene triangles that have one congruent angle that measures 100° are similar.

Rationale for Option D: This is incorrect. The student may have incorrectly concluded that all isosceles triangles with one angle of 40° should have another base angle of 40° . However, an isosceles triangle could have a vertex angle of 40° , which leaves its base angle measurements as 70° and 70° . This triangle would not be similar to triangle XYZ. A similar triangle would have to have both base angles be 40° not just one of them.

Sample Response: 1 point

Triangle XYZ is shown.



Which triangle must be similar to triangle XYZ?

- ☒ a triangle with two angles that measure 40°
- ☐ (B) a triangle with angles that measure 40° and 60°
- ☐ (C) a scalene triangle with only one angle that measures 100°
- ☐ (D) an isosceles triangle with only one angle that measures 40°

Geometry
Spring 2017 Item Release

Question 4

Question and Scoring Guidelines

Question 4

During a 90-day semester, a student records whether he arrives at school on time and whether he goes to bed by 10:00 p.m. the night before. The results are shown in the table.

Number of Occurrences		
	Arrives at School on Time	Arrives at School Late
Goes to Bed by 10:00 p.m.	72	8
Goes to Bed After 10:00 p.m.	9	1

Does the student arriving at school on time depend on whether the student goes to bed by 10:00 p.m.? Justify your reasoning.

Type your answer in the space provided.

B *I* U $\mathcal{I}_x$ $\frac{1}{2}$ $\frac{3}{4}$ $\frac{5}{8}$ $\frac{7}{8}$ ✂ 📄 📋 ↶ ↷ 🔍 Ω $f(x)$

Words 0/4000, # Chars 0/20000

Points Possible: 2

Content Cluster: Understand independence and conditional probability, and use them to interpret data.

Content Standard: Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. *For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer. (S.CP.5)*

Scoring Guidelines

Score Point

2 points

Description

The response includes the following correct Statement 1 with a correct Justification:

Statement 1:

- a) No, the events are independent.

Justification:

- a) The probability of the student arriving late given that he or she goes to bed by 10:00 p.m. ($\frac{8}{80}$) is equal to the probability that the student arrives late given that he or she goes to bed after 10:00 p.m. ($\frac{1}{10}$), so the two events are independent of each other.

OR

- b) The probability of going to bed by 10:00 p.m. is $\frac{80}{90}$. The probability of arriving to school on time is $\frac{81}{90}$. Since the probability of doing both is ($\frac{72}{90} = \frac{80}{90} \bullet \frac{81}{90}$), the two events are independent.

OR

- c) Given that the student went to bed by 10:00 p.m., the ratio of the number of occurrences of the student arriving at the school late to the number of occurrences that the student arrives at school on time is 8 to 72. Given that student went to bed after 10:00 p.m., the ratio of the number of occurrences of the student arriving at school late to the number of occurrences of student arriving at school on time is 1 to 9. Since 8 to 72 is equivalent to 1 to 9, the events are independent.

1 point

The response includes the correct Statement 1 listed above with a partially correct Justification.

0 points

The response does not meet the criteria required to earn one point. The response indicates inadequate or no understanding of the task and/or the idea or concept needed to answer the item. It may only repeat information given in the test item. The response may provide an incorrect solution/response and the provided supportive information may be irrelevant to the item, or possibly, no other information is shown. The student may have written on a different topic or written, "I don't know."

Geometry
Spring 2017 Item Release

Question 4

Sample Responses

Sample Response: 2 points

During a 90-day semester, a student records whether he arrives at school on time and whether he goes to bed by 10:00 p.m. the night before. The results are shown in the table.

Number of Occurrences		
	Arrives at School on Time	Arrives at School Late
Goes to Bed by 10:00 p.m.	72	8
Goes to Bed After 10:00 p.m.	9	1

Does the student arriving at school on time depend on whether the student goes to bed by 10:00 p.m.? Justify your reasoning.

Type your answer in the space provided.

B *I* U *I_x*

No, it doesn't because based on the information provided, the probability that the child will show up when he or she goes to bed before 10:00 is 90%. But if the student doesn't go to bed at 10:00, they still arrive to school on time 90%, meaning that the time that this student goes to bed doesn't have any effect on their attendance.

Words 63/4000, # Chars 333/20000

Notes on Scoring

This response earns full credit (2 points) because it shows the correct statement ("No, it doesn't...") and the correct justification for the statement.

The two events are independent if their probabilities are equal or if the probability of them occurring together is the product of their individual probabilities.

The probability that the student arrives late given that he or she goes to bed by 10:00 p.m. is $\frac{8}{(72+8)}$ or $\frac{8}{80}$ or $\frac{1}{10}$. The probability that the student arrives late given that he or she goes to bed after 10:00 p.m. is $\frac{1}{(9+1)}$ or $\frac{1}{10}$. Since two probabilities are equal, $(\frac{8}{80} = \frac{1}{10})$, the two events are independent.

Similarly, the probability that the student arrives on time given that he or she goes to bed by 10:00 p.m. is $\frac{72}{80}$ or 90%. The probability that the student arrives on time given that he or she goes to bed after 10:00 p.m. is $\frac{9}{10}$ or 90%. Since two probabilities are equal, the events are independent.

Also, the probability of going to bed by 10:00 pm is $\frac{(72+8)}{90}$ or $\frac{80}{90}$. The probability of arriving to school on time is $\frac{(72+9)}{90}$ or $\frac{81}{90}$. According to the table, the probability of going to bed and arriving at school on time is approximately $\frac{72}{90}$. Since $\frac{(72+9)}{90} \cdot \frac{81}{90}$ equals to $(\frac{72}{90})$, the two events are independent.

The ratio of the number of times the student arrives late to the number of times he or she arrives on time, given that the student goes to bed by 10:00 p.m., is $\frac{8}{72}$. The ratio of the number of times the student arrives late to the number of times he or she arrives on time, given that the student goes to bed after 10:00 p.m., is $\frac{1}{9}$. Since both ratios are equal and $(\frac{8}{72} = \frac{1}{9})$, the two events are independent.

Sample Response: 2 points

During a 90-day semester, a student records whether he arrives at school on time and whether he goes to bed by 10:00 p.m. the night before. The results are shown in the table.

	Number of Occurrences	
	Arrives at School on Time	Arrives at School Late
Goes to Bed by 10:00 p.m.	72	8
Goes to Bed After 10:00 p.m.	9	1

Does the student arriving at school on time depend on whether the student goes to bed by 10:00 p.m.? Justify your reasoning.

Type your answer in the space provided.

B I U T

No, arriving at school on time is not affected by what time the student goes to bed. When he goes to bed on time he was still late 1/10 of the time, the same chances of being late when he did go to bed after 10.

Words 46/4000, # Chars 212/20000

Notes on Scoring

This response earns full credit (2 points) because it shows the correct statement (“No, arriving at school on time is not affected by what time the student goes to bed.”) and the correct justification for the statement.

The two events are independent if their probabilities are equal or if the probability of them occurring together is the product of their probabilities.

The probability that the student arrives late given that he or she goes to bed by 10:00 p.m. is $\frac{8}{72+8}$ or $\frac{8}{80}$ or $\frac{1}{10}$. The probability that the student arrives late given that he goes to bed after 10:00 p.m. is $\frac{1}{9+1}$ or $\frac{1}{10}$. Since the two probabilities are equal, $(\frac{8}{80} = \frac{1}{10})$, the two events are independent.

Sample Response: 1 point

During a 90-day semester, a student records whether he arrives at school on time and whether he goes to bed by 10:00 p.m. the night before. The results are shown in the table.

Number of Occurrences		
	Arrives at School on Time	Arrives at School Late
Goes to Bed by 10:00 p.m.	72	8
Goes to Bed After 10:00 p.m.	9	1

Does the student arriving at school on time depend on whether the student goes to bed by 10:00 p.m.? Justify your reasoning.

Type your answer in the space provided.

B *I* U ~~I_x~~

No because once every nine times is very similar to 8 times every 72 times

Words 15/4000, # Chars 74/20000

Notes on Scoring

This response earns partial credit (1 point) because it shows the correct statement ("No because...") but provides an incomplete justification for the statement.

Sample Response: 1 point

During a 90-day semester, a student records whether he arrives at school on time and whether he goes to bed by 10:00 p.m. the night before. The results are shown in the table.

Number of Occurrences		
	Arrives at School on Time	Arrives at School Late
Goes to Bed by 10:00 p.m.	72	8
Goes to Bed After 10:00 p.m.	9	1

Does the student arriving at school on time depend on whether the student goes to bed by 10:00 p.m.? Justify your reasoning.

Type your answer in the space provided.

B *I* U *I_x*

No it does not, it is the exact same chance that they will end up late to school

Words 18/4000, # Chars 80/20000

Notes on Scoring

This response earns partial credit (1 point) because it shows the correct statement ("No it does not...") but provides an incomplete justification for the statement.

Sample Response: 1 point

During a 90-day semester, a student records whether he arrives at school on time and whether he goes to bed by 10:00 p.m. the night before. The results are shown in the table.

Number of Occurrences		
	Arrives at School on Time	Arrives at School Late
Goes to Bed by 10:00 p.m.	72	8
Goes to Bed After 10:00 p.m.	9	1

Does the student arriving at school on time depend on whether the student goes to bed by 10:00 p.m.? Justify your reasoning.

Type your answer in the space provided.

B *I* U ~~Strikethrough~~

☰ ☷ ☹ ☺

✂ ☰ ☷ ☹ ☺

↶ ↷

🔍 Ω f(x)

No the student arriving on time does not depend on whether or not the student goes to bed by 10 p.m. The student arrived at school late just as many times as when he went to bed at 10 p.m.

Words 40/4000, # Chars 188/20000

Notes on Scoring

This response earns partial credit (1 point) because it shows the correct statement ("No...") but provides an incorrect justification for the statement. The student may have confused the number of occurrences of the two events with the ratio of the events.

Sample Response: 1 point

During a 90-day semester, a student records whether he arrives at school on time and whether he goes to bed by 10:00 p.m. the night before. The results are shown in the table.

	Number of Occurrences	
	Arrives at School on Time	Arrives at School Late
Goes to Bed by 10:00 p.m.	72	8
Goes to Bed After 10:00 p.m.	9	1

Does the student arriving at school on time depend on whether the student goes to bed by 10:00 p.m.? Justify your reasoning.

Type your answer in the space provided.

It does not depend on if the student goes to bed at 10 p.m. or not. In the data shown the student does get to school on time when he goes to bed at 10 p.m., but she also gets to school on time when he goes to bed after that time. Also, when the student goes to bed after 10 p.m., it seems that he was not late to school as much if he went to bed at 10 p.m. Therefore, it does not matter on what time the student gets to bed.

Notes on Scoring

This response earns partial credit (1 point) because it shows the correct statement ("It does not depend on if the student goes to bed at 10 p.m. or not.") but provides a statistically incorrect justification for the statement.

Sample Response: 0 points

During a 90-day semester, a student records whether he arrives at school on time and whether he goes to bed by 10:00 p.m. the night before. The results are shown in the table.

	Number of Occurrences	
	Arrives at School on Time	Arrives at School Late
Goes to Bed by 10:00 p.m.	72	8
Goes to Bed After 10:00 p.m.	9	1

Does the student arriving at school on time depend on whether the student goes to bed by 10:00 p.m.? Justify your reasoning.

Type your answer in the space provided.

If the student goes to bed by 10, then the probability of him getting to school on time is much greater than him going to bed after 10. I know this because when the student went to bed before 10, the table tells me he arrived at school on time 72 times. But when he didn't go to bed until after 10, he only arrived to school early 9 times. The probability of him making it to school on time is higher if he goes to bed a lot sooner.

Notes on Scoring

This response earns no credit (0 points) because it shows an incorrect statement and provides a statistically incorrect justification for the situation.

Sample Response: 0 points

During a 90-day semester, a student records whether he arrives at school on time and whether he goes to bed by 10:00 p.m. the night before. The results are shown in the table.

Number of Occurrences		
	Arrives at School on Time	Arrives at School Late
Goes to Bed by 10:00 p.m.	72	8
Goes to Bed After 10:00 p.m.	9	1

Does the student arriving at school on time depend on whether the student goes to bed by 10:00 p.m.? Justify your reasoning.

Type your answer in the space provided.

B *I* U *I_x*

Yes and no because everyone is different sometimes its harder to get up in the morning then for others. But since this person is doing this survey he was only late once going to bed after 10. So in this case for this person no it does not matter.

Words 49/4000, # Chars 246/20000

Notes on Scoring

This response earns no credit (0 points) because it shows an incorrect statement based on an incorrect justification.

Sample Response: 0 points

During a 90-day semester, a student records whether he arrives at school on time and whether he goes to bed by 10:00 p.m. the night before. The results are shown in the table.

Number of Occurrences		
	Arrives at School on Time	Arrives at School Late
Goes to Bed by 10:00 p.m.	72	8
Goes to Bed After 10:00 p.m.	9	1

Does the student arriving at school on time depend on whether the student goes to bed by 10:00 p.m.? Justify your reasoning.

Type your answer in the space provided.

B *I* U ~~ABC~~

Yes, the student arriving on time does depend on whether the student goes to bed by 10:00 p.m., because in the students records he arrives to school ontime 72 times, but is late 8 time arriving. Him not going to bed on time will be 9 times he is on time to school and 1 time he is late arriving to school. So, in this situation he should go to bed around 9, then will be on time, and not be late.

Words 82/4000, # Chars 396/20000

Notes on Scoring

This response earns no credit (0 points) because it shows an incorrect statement based on incorrect justification.

Geometry
Spring 2017 Item Release

Question 5

Question and Scoring Guidelines

Question 5

A globe has a diameter of 12 inches. It fits inside a cube-shaped box that has a side length of 12 inches.

What is the volume, rounded to the nearest hundredth of a cubic inch, of the space inside the box that is not taken up by the globe?

cubic inches

A digital calculator interface with a numeric keypad on the left and a large display area on the right. The keypad includes buttons for digits 1-9, 0, a decimal point, a negative sign, and a fraction template icon. Above the keypad are navigation buttons: left arrow, right arrow, undo, redo, and a clear button.

Points Possible: 1

Content Cluster: Explain volume formulas, and use them to solve problems.

Content Standard: Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems. (G.GMD.3)

Scoring Guidelines

Exemplar Response

- 823.22

Other Correct Responses

- Any value between 822.857 and 823.68, inclusive.

For this item, a full-credit response includes:

- A correct volume (1 point).

Geometry
Spring 2017 Item Release

Question 5

Sample Responses

Sample Response: 1 point

A globe has a diameter of 12 inches. It fits inside a cube-shaped box that has a side length of 12 inches.

What is the volume, rounded to the nearest hundredth of a cubic inch, of the space inside the box that is not taken up by the globe?

823.22

cubic inches



1	2	3
4	5	6
7	8	9
	0	
.	-	$\frac{\Box}{\Box}$

Notes on Scoring

This response earns full credit (1 point) because it shows a correct answer for the space inside the box that is not taken up by the globe, rounded to the nearest hundredth of a cubic inch. In this situation, the calculation can be obtained by finding the difference between the volume of the cube, $12^3 = 1728$ *cubic inches*, and the volume of the sphere (globe), $\frac{4}{3}\pi r^3$. Since the sphere has a diameter of 12 inches, its radius is 6 inches and the volume is $\frac{4}{3}\pi \cdot 6^3 \approx 904.7786832....$

The difference of volumes is

$$1728 - 904.7786832....$$

$$\approx 823.2213158.$$

When rounded to the nearest hundredths, the answer is 823.22 cubic inches. Answers between 822.857 and 823.68 are accepted to allow for minor differences in rounding.

Sample Response: 1 point

A globe has a diameter of 12 inches. It fits inside a cube-shaped box that has a side length of 12 inches.

What is the volume, rounded to the nearest hundredth of a cubic inch, of the space inside the box that is not taken up by the globe?

823.68

cubic inches

A digital math input interface. At the top, there are five navigation buttons: left arrow, right arrow, undo, redo, and a clear button (X). Below these is a numeric keypad with buttons for digits 1 through 9, 0, a decimal point, a negative sign, and a fraction template button. The keypad is currently empty.

Notes on Scoring

This response earns full credit (1 point) because it shows an answer that falls into the correct range between 822.857 and 823.68 of the accepted values for the space inside the box that is not taken up by the globe (to allow for minor differences in rounding).

In this situation, the calculation was obtained using 3.14 for pi and by finding the difference between the volume of the cube, $12^3 = 1728$ *cubic inches*, and the volume of the sphere (globe), $\frac{4}{3}\pi r^3$. Since the student used 3.14 for pi, the globe has a diameter of 12 inches, its radius is 6 inches and the volume is $\frac{4}{3}((3.14) \cdot 6^3) \approx 904.32$.

The difference of volumes is $1728 - 904.32 = 823.68$. The answer of 823.68 cubic inches falls within the accepted range.

Sample Response: 0 points

A globe has a diameter of 12 inches. It fits inside a cube-shaped box that has a side length of 12 inches.

What is the volume, rounded to the nearest hundredth of a cubic inch, of the space inside the box that is not taken up by the globe?

5510.23

cubic inches

A digital calculator interface is shown. At the top, there is a row of five navigation icons: a left arrow, a right arrow, a double left arrow, a double right arrow, and a close button (an 'X' in a square). Below this is a numeric keypad with three rows of buttons. The first row contains '1', '2', and '3'. The second row contains '4', '5', and '6'. The third row contains '7', '8', and '9'. The fourth row contains a decimal point button (a dot in a square), a zero button ('0' in a square), and a fraction button (a square with a horizontal line and two smaller squares above and below). The fifth row contains a minus sign button ('-' in a square), a plus sign button ('+' in a square), and a divide button (a square with a horizontal line and a smaller square below). To the right of the keypad is a large, empty rectangular display area.

Notes on Scoring

This response earns no credit (0 points) because it shows an incorrect answer that is out of the correct range of accepted values.

This student may have used the diameter of 12 instead of the radius of 6 when doing calculations. Then he or she may have taken the difference of the sphere and the cube without realizing the cube was larger. His or her calculation of the sphere may have been $3\pi \cdot 12^3 \approx 7238.229474...$ and then he or she subtracted the cube from the sphere to get $7238.229474 - 1728 \approx 5510.23$ when rounded to the nearest hundredths.

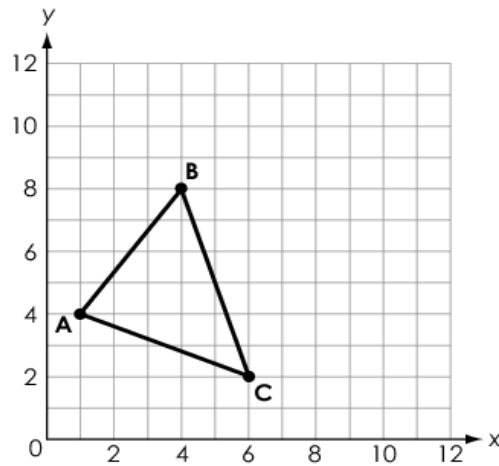
Geometry
Spring 2017 Item Release

Question 6

Question and Scoring Guidelines

Question 6

Triangle ABC is shown.



Triangle $A'B'C'$ is created by dilating triangle ABC by a scale factor of 4.

What is the length of $\overline{A'B'}$?

←
→
↶
↷
✖

1	2	3	+	-	•	÷					
4	5	6	<	≤	=	≥	>				
7	8	9	$\frac{\Box}{\Box}$	$\Box^\Box$	$\Box_\Box$	()		$\sqrt{\Box}$	$\sqrt[\Box]{\Box}$	π	i
0	.	-	sin	cos	tan	arcsin	arccos	arctan			

Points Possible: 1

Content Cluster: Understand similarity in terms of similarity transformations.

Content Standard: Verify experimentally the properties of dilations given by a center and a scale factor. (*G.SRT.1*)
b. The dilation of a line segment is longer or shorter in the ratio given by the scale factor.

Scoring Guidelines

Exemplar Response

- 20

Other Correct Responses

- Any equivalent value

For this item, a full-credit response includes:

- A correct length (1 point).

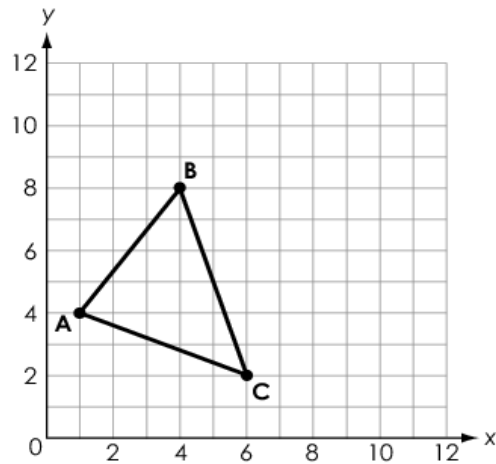
Geometry
Spring 2017 Item Release

Question 6

Sample Responses

Sample Response: 1 point

Triangle ABC is shown.



Triangle A'B'C' is created by dilating triangle ABC by a scale factor of 4.

What is the length of $\overline{A'B'}$?

20



1	2	3	+	-	•	÷					
4	5	6	<	≤	=	≥	>				
7	8	9	$\frac{\Box}{\Box}$	$\Box^\Box$	$\Box_\Box$	()		$\sqrt{\Box}$	$\sqrt[\Box]{\Box}$	π	i
0	.	-	sin	cos	tan	arcsin	arccos	arctan			

Notes on Scoring

This response earns full credit (1 point) because it shows the correct length of side A'B'.

When a triangle is dilated by a positive scale factor of 4, all side lengths are getting changed by this scale factor, regardless of the dilation center. The distance formula and coordinates of points A(1, 4) and B(4, 8) can be used to calculate the length of side AB as

$$\sqrt{((4 - 1)^2 + (8 - 4)^2)}$$

$$= \sqrt{(3^2 + 4^2)}$$

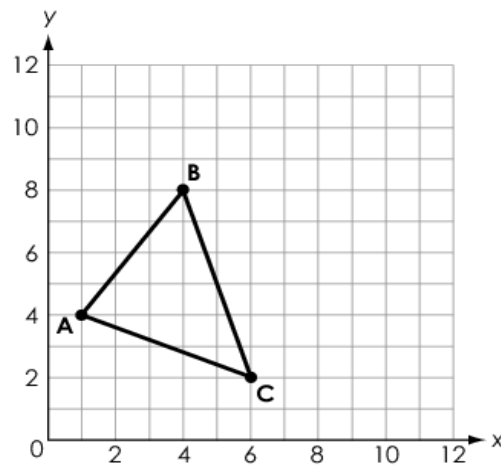
$$= \sqrt{25}$$

$$= 5.$$

By applying the scale factor 4 to the length of side AB, the length of side A'B' = $4 \bullet 5 = 20$ units.

Sample Response: 1 point

Triangle ABC is shown.



Triangle A'B'C' is created by dilating triangle ABC by a scale factor of 4.

What is the length of $\overline{A'B'}$?

$$4\sqrt{25}$$

←	→	↶	↷	✖							
1	2	3	+	-	•	÷					
4	5	6	<	≤	=	≥	>				
7	8	9	$\frac{\Box}{\Box}$	$\Box^\Box$	$\Box_\Box$	()		$\sqrt{\Box}$	$\sqrt[\Box]{\Box}$	π	i
0	.	-	sin	cos	tan	arcsin	arccos	arctan			

Notes on Scoring

This response earns full credit (1 point) because it shows an equivalent value ($4\sqrt{25} = 4 \cdot 5 = 20$) for the correct length of side A'B'.

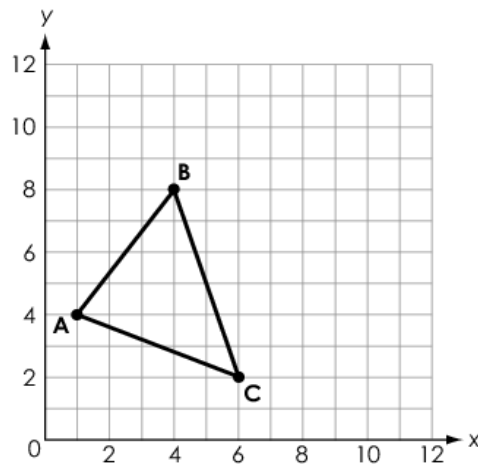
When a triangle is dilated by a positive scale factor of 4, all side lengths are getting changed by this scale factor, regardless of the dilation center. The distance formula and coordinates of points A(1, 4) and B(4, 8) can be used to calculate the length of side AB as

$$\begin{aligned} & \sqrt{((4 - 1)^2 + (8 - 4)^2)} \\ &= \sqrt{(3^2 + 4^2)} \\ &= \sqrt{25} \end{aligned}$$

By applying the scale factor of 4 to the length of side AB, the length of side A'B' = $4 \cdot \sqrt{25}$

Sample Response: 0 points

Triangle ABC is shown.



Triangle A'B'C' is created by dilating triangle ABC by a scale factor of 4.

What is the length of $\overline{A'B'}$?

5



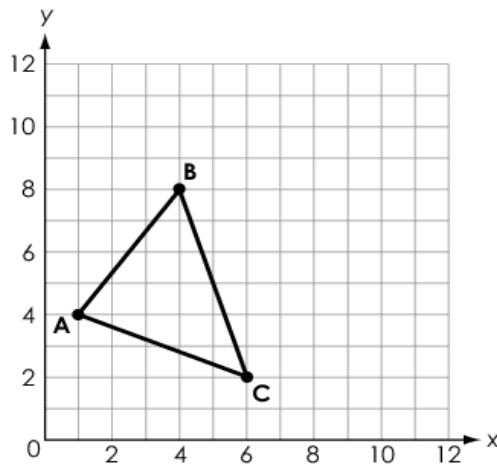
1	2	3	+	-	•	÷					
4	5	6	<	≤	=	≥	>				
7	8	9	$\frac{\Box}{\Box}$	$\Box^\Box$	$\Box_\Box$	()		$\sqrt{\Box}$	$\sqrt[\Box]{\Box}$	π	i
0	.	-	sin	cos	tan	arcsin	arccos	arctan			

Notes on Scoring

This response earns no credit (0 points) because it shows an incorrect answer for the length of side $A'B'$. The student may have found the length of side AB instead of the dilated side $A'B'$.

Sample Response: 0 points

Triangle ABC is shown.



Triangle A'B'C' is created by dilating triangle ABC by a scale factor of 4.

What is the length of $\overline{A'B'}$?

52

Calculator interface showing the input 52 and a keypad with various mathematical symbols and functions.

Notes on Scoring

This response earns no credit (0 points) because it shows an incorrect answer for the length of side A'B'.

The student may have dilated the line segment AB about the origin to get A'B' with end points at A' (4, 16) and B' (16, 32), and then incorrectly used the distance formula as

$$\begin{aligned} & \sqrt{((4 + 16)^2 + (16 + 32)^2)} \\ &= \sqrt{2704} \\ &= 52 \end{aligned}$$

Geometry
Spring 2017 Item Release

Question 7

Question and Scoring Guidelines

Question 7

A study reports that in 2010 the population of the United States was 308,745,538 people and the land area was approximately 3,531,905 square miles.

Based on the study, what was the population density, in people per square mile, of the United States in 2010? Round your answer to the nearest tenth.

people per square mile

←

→

↶

↷

✖

1	2	3
4	5	6
7	8	9
	0	
.	-	$\frac{\Box}{\Box}$

Points Possible: 1

Content Cluster: Apply geometric concepts in modeling situations.

Content Standard: Apply concepts of density based on area and volume in modeling situations, e.g., persons per square mile, BTUs per cubic foot. (G.MG.2)

Scoring Guidelines

Exemplar Response

- 87.4

Other Correct Responses

- Any value between 87.4 and 87.42, inclusive

For this item, a full-credit response includes:

- A correct density (1 point)

Geometry
Spring 2017 Item Release

Question 7

Sample Responses

Sample Response: 1 point

A study reports that in 2010 the population of the United States was 308,745,538 people and the land area was approximately 3,531,905 square miles.

Based on the study, what was the population density, in people per square mile, of the United States in 2010? Round your answer to the nearest tenth.

87.4

people per square mile

A digital calculator interface. At the top, a text input field contains the number '87.4'. To the right of this field, the text 'people per square mile' is written in an italicized font. Below the text input is a standard calculator keypad with buttons for digits 1-9, 0, a decimal point, a negative sign, and a fraction template button. Above the keypad is a row of navigation buttons: left arrow, right arrow, undo, redo, and a delete/clear button.

Notes on Scoring

This response earns full credit (1 point) because it shows an answer that falls into the correct range of the accepted values for the population density, rounded to the nearest tenth (to allow for minor rounding errors).

The population density is a measure that expresses the number of people per square unit of area. In this situation, the population density of the United States, or the number of people per square mile, is the ratio of 308,745,538 people to the area of 3,531,905 square miles, or $308,745,538 / 3,531,905 \approx 87.41615021 \dots$ or 87.4 people per square miles, when rounded to the nearest tenth.

Sample Response: 1 point

A study reports that in 2010 the population of the United States was 308,745,538 people and the land area was approximately 3,531,905 square miles.

Based on the study, what was the population density, in people per square mile, of the United States in 2010? Round your answer to the nearest tenth.

87.4162

people per square mile

A digital calculator interface. At the top, a display shows the number 87.4162. Below the display is a row of navigation buttons: left arrow, right arrow, undo, redo, and a clear button (X). Below these is a numeric keypad with buttons for digits 1-9, 0, a decimal point, a negative sign, and a fraction template icon.

Notes on Scoring

This response earns full credit (1 point) because it shows an answer that falls into the correct range of the accepted values for the population density, rounded to the nearest tenth (to allow for minor rounding errors).

The population density is a measure that expresses the number of people per square unit of area. In this situation, the population density of the United States, or the number of people per square mile, is the ratio of 308,745,538 people to the area of 3,531,905 square miles, or $308,745,538/3,531,905 \approx 87.41615021...$ or 87.4162 people per square miles, when rounded to the nearest ten thousandth.

Sample Response: 0 points

A study reports that in 2010 the population of the United States was 308,745,538 people and the land area was approximately 3,531,905 square miles.

Based on the study, what was the population density, in people per square mile, of the United States in 2010? Round your answer to the nearest tenth.

87

people per square mile

A digital calculator interface. At the top, there is a row of five navigation buttons: left arrow, right arrow, undo, redo, and a close button (X). Below this is a numeric keypad with buttons for digits 1 through 9, 0, a decimal point (.), a negative sign (-), and a fraction template button (a box with a horizontal line and two smaller boxes above and below). To the right of the keypad is a large, empty rectangular display area.

Notes on Scoring

This response earns no credit (0 points) because it shows an incorrect answer that does not fall into the correct range of accepted values for the population density.

Sample Response: 0 points

A study reports that in 2010 the population of the United States was 308,745,538 people and the land area was approximately 3,531,905 square miles.

Based on the study, what was the population density, in people per square mile, of the United States in 2010? Round your answer to the nearest tenth.

0.01144

people per square mile

A digital calculator interface with a light gray background. At the top, there is a row of five circular navigation buttons: a left arrow, a right arrow, a double left arrow, a double right arrow, and a button with an 'X' icon. Below these buttons is a numeric keypad with a grid of buttons. The first row contains buttons for digits 1, 2, and 3. The second row contains buttons for digits 4, 5, and 6. The third row contains buttons for digits 7, 8, and 9. The fourth row contains buttons for 0, a decimal point (.), a minus sign (-), and a fraction template icon (a box with a horizontal line and two smaller boxes below it). The keypad is positioned on the left side of a larger white rectangular area.

Notes on Scoring

This response earns no credit (0 points) because it shows an incorrect answer that does not fall into the correct range of accepted values for the population density.

The student may have reversed the ratio. Instead of comparing the people to the area, he or she may have compared the area to the people, such as $\frac{31,905}{308,745,538}$, which is 0.01144.

Geometry
Spring 2017 Item Release

Question 8

Question and Scoring Guidelines

Question 8

A figure is fully contained in Quadrant II. The figure is transformed as shown.

- a reflection over the x -axis
- a reflection over the line $y = x$
- a 90° counterclockwise rotation about the origin

In which quadrant does the resulting image lie?

- Ⓐ Quadrant I
- Ⓑ Quadrant II
- Ⓒ Quadrant III
- Ⓓ Quadrant IV

Points Possible: 1

Content Cluster: Understand congruence in terms of rigid motions.

Content Standard: Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent. (G.CO.6)

Scoring Guidelines

Rationale for Option A: This is incorrect. The student may have reflected the figure over the y -axis instead of the line $y = x$.

Rationale for Option B: This is incorrect. The student may have reflected the figure over the line $y = -x$ instead of $y = x$.

Rationale for Option C: This is incorrect. The student may have reflected the figure over the y -axis instead of the line $y = x$ and rotated the figure clockwise instead of counterclockwise.

Rationale for Option D: **Key** – The student correctly noticed that a reflection over the x -axis will reflect a figure in Quadrant II into Quadrant III. A reflection across the line $y = x$ will keep the figure in Quadrant III. A counterclockwise rotation about the origin will move the figure into Quadrant IV.

Sample Response: 1 point

A figure is fully contained in Quadrant II. The figure is transformed as shown.

- a reflection over the x -axis
- a reflection over the line $y = x$
- a 90° counterclockwise rotation about the origin

In which quadrant does the resulting image lie?

- ☐ (A) Quadrant I
- ☐ (B) Quadrant II
- ☐ (C) Quadrant III
- ☒ Quadrant IV

Geometry
Spring 2017 Item Release

Question 9

Question and Scoring Guidelines

Question 9

An online retailer conducts a random survey of its customers. The survey shows that 80% of the customers receive their purchases within four days, and 95% of those customers are satisfied with the quality of their purchases.

What percent of all customers receive their purchases within four days and are not satisfied with the quality of their purchases?

- ☐ (A) 4%
- ☐ (B) 5%
- ☐ (C) 19%
- ☐ (D) 24%

Points Possible: 1

Content Cluster: Use the rules of probability to compute probabilities of compound events in a uniform probability model.

Content Standard: Find the conditional probability of A given B as the fraction of B's outcomes that also belong to A, and interpret the answer in terms of the model. (*S.CP.6*)

Scoring Guidelines

Rationale for Option A: Key – The student correctly concluded that 5% of the customers who received their purchases within four days were not satisfied ($100 - 95 = 5\%$), and 5% of 80% of all customers who received their purchases within four days is $0.05 \bullet 80 = 4\%$.

Rationale for Option B: This is incorrect. The student may have found that only 5% of the customers who received their purchases within four days were not satisfied, and did not multiply 5% by 80% to find the percentage of all customers.

Rationale for Option C: This is incorrect. The student may have reversed the conditional probability, found the complement of the customers who received their purchases within four days is 20% and then found 95% of 20% as $0.95 \bullet 0.20 = 0.19$ or 19%.

Rationale for Option D: This is incorrect. The student may have found the percentage of all customers who received their purchases within four days and were satisfied with the purchases by multiplying 95% by 80%, or $0.95 \bullet 0.80 = 0.76$ or 76%, and then subtracted that from 100% to get 24%.

Sample Response: 1 point

An online retailer conducts a random survey of its customers. The survey shows that 80% of the customers receive their purchases within four days, and 95% of those customers are satisfied with the quality of their purchases.

What percent of all customers receive their purchases within four days and are not satisfied with the quality of their purchases?

- ☒ 4%
- ☐ (B) 5%
- ☐ (C) 19%
- ☐ (D) 24%

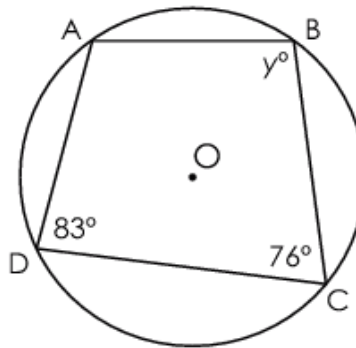
Geometry
Spring 2017 Item Release

Question 10

Question and Scoring Guidelines

Question 10

Quadrilateral ABCD is inscribed in circle O, as shown.



What is the value of y ?

$y =$

←

→

↶

↷

✖

1	2	3
4	5	6
7	8	9
	0	
.	-	$\frac{\Box}{\Box}$

Points Possible: 1

Content Cluster: Understand and apply theorems about circles.

Content Standard: Construct the inscribed and circumscribed circles of a triangle; prove and apply the property that opposite angles are supplementary for a quadrilateral inscribed in a circle. (G.C.3)

Scoring Guidelines

Exemplar Response

- $y = 97$

Other Correct Responses

- Any equivalent value

For this item, a full-credit response includes:

- The correct value (1 point).

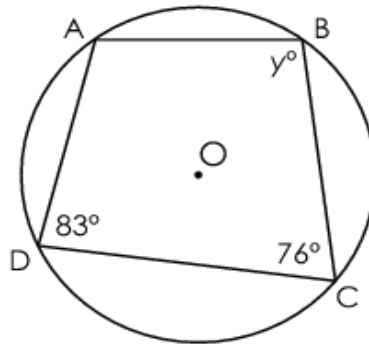
Geometry
Spring 2017 Item Release

Question 10

Sample Responses

Sample Response: 1 point

Quadrilateral ABCD is inscribed in circle O, as shown.



What is the value of y ?

$y =$

Calculator interface showing navigation buttons (left, right, undo, redo, delete) and a numeric keypad with digits 1-9, 0, a decimal point, a negative sign, and a fraction template button.

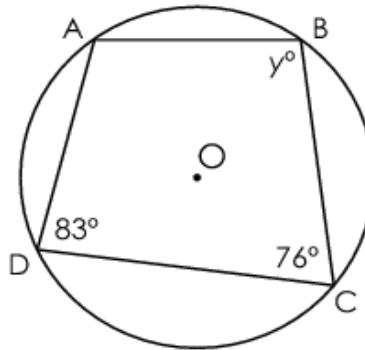
Notes on Scoring

This response earns full credit (1 point) because it shows the correct value for y .

The solution is based on the theorem that opposite angles of a quadrilateral inscribed in a circle are supplementary, or have a sum of 180° . Using this fact, $y + 83 = 180$ and $y = 97^\circ$.

Sample Response: 1 point

Quadrilateral ABCD is inscribed in circle O, as shown.



What is the value of y ?

$y =$

←

→

↶

↷

✖

1	2	3
4	5	6
7	8	9
	0	
.	-	$\frac{\Box}{\Box}$

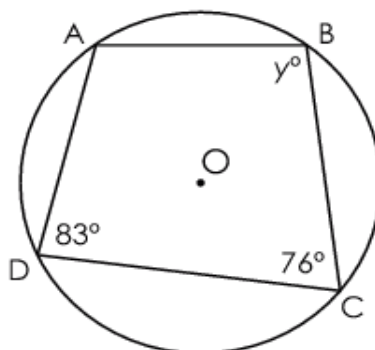
Notes on Scoring

This response earns full credit (1 point) because it shows the correct value for y .

The solution is based on the theorem that opposite angles of a quadrilateral inscribed in a circle are supplementary, or have a sum of 180° . Using this fact, $y + 83 = 180$ and $y = 97^\circ$, which equals 97.0° .

Sample Response: 0 points

Quadrilateral ABCD is inscribed in circle O, as shown.



What is the value of y ?

$y =$

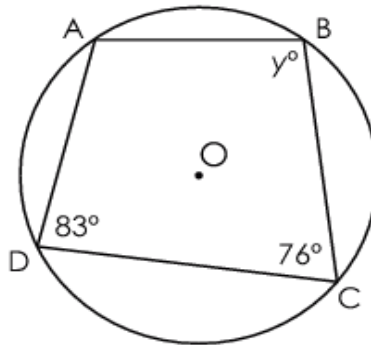
Calculator interface showing a numeric keypad and a display area. The display area is empty.

Notes on Scoring

This response earns no credit (0 points) because it shows an incorrect value for y . The student may have incorrectly thought that the measures of opposite angles of a quadrilateral inscribed in a circle are equal.

Sample Response: 0 points

Quadrilateral ABCD is inscribed in circle O, as shown.



What is the value of y ?

$y =$

Calculator interface showing a numeric keypad and a display area. The keypad includes buttons for digits 1-9, 0, a decimal point, a negative sign, and a fraction template button. The display area is currently empty.

Notes on Scoring

This response earns no credit (0 points) because it shows an incorrect value for y . The student may have incorrectly thought that angle y is a right angle which measure 90° .

Geometry
Spring 2017 Item Release

Question 11

Question and Scoring Guidelines

Question 11

Josh has a bag containing pieces of candy. The bag contains 10 red circular pieces, 10 red square pieces, 10 blue triangular pieces, and 10 blue star-shaped pieces. He draws a red piece of candy from the bag.

What is the complement of this event?

- Ⓐ He draws a blue piece.
- Ⓑ He draws a square piece.
- Ⓒ He draws a circular piece.
- Ⓓ He draws a star-shaped piece.

Points Possible: 1

Content Cluster: Understand independence and conditional probability, and use them to interpret data.

Content Standard: Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not"). (*S.CP.1*)

Scoring Guidelines

Rationale for Option A: **Key** – The student correctly identified that the complement of drawing a red piece is drawing a blue piece since all of the candy can be described as either being red or blue.

Rationale for Option B: This is incorrect. The student may have incorrectly thought complement means to describe the candy using a secondary feature and chose one of the shapes that describes some of the red candies.

Rationale for Option C: This is incorrect. The student may have incorrectly thought complement means to describe the candy using a secondary feature and chose one of the shapes that describes some of the red candies.

Rationale for Option D: This is incorrect. The student may have correctly realized that the complement of drawing a red candy has to be a descriptor that cannot describe any red candy, but the student incorrectly chose the shape of a blue candy. The star-shaped candy is not the complement. It is only a part of all the blue candies, and therefore it is only one part of the complement instead of the entire complement.

Sample Response: 1 point

Josh has a bag containing pieces of candy. The bag contains 10 red circular pieces, 10 red square pieces, 10 blue triangular pieces, and 10 blue star-shaped pieces. He draws a red piece of candy from the bag.

What is the complement of this event?

- ☒ He draws a blue piece.
- ☐ He draws a square piece.
- ☐ He draws a circular piece.
- ☐ He draws a star-shaped piece.

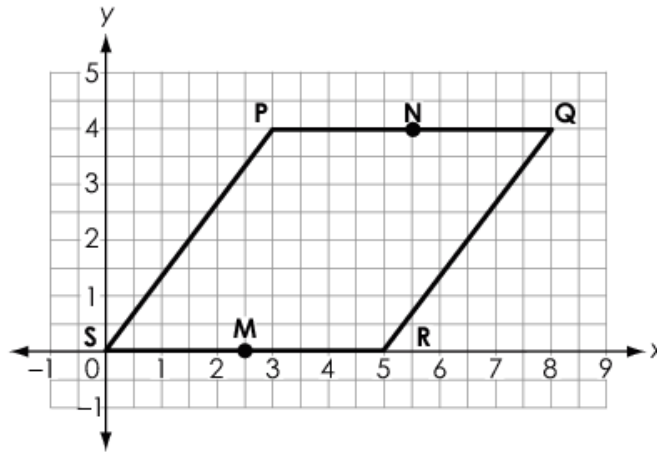
Geometry
Spring 2017 Item Release

Question 12

Question and Scoring Guidelines

Question 12

Rhombus PQRS is shown on the coordinate plane. Points M and N are midpoints of their respective sides.



Select all of the transformations that map the rhombus onto itself.

- ☐ a 90° clockwise rotation around the center of the rhombus
- ☐ a 180° clockwise rotation around the center of the rhombus
- ☐ a reflection across $\overline{PR}$
- ☐ a reflection across $\overline{NM}$
- ☐ a reflection across $\overline{QS}$

Points Possible: 1

Content Strand: Experiment with transformation in the plane.

Content Standard: Identify the symmetries of a figure, which are the rotations and reflections that carry it onto itself. (G.CO.3)

Scoring Guidelines

Rationale for First Option: This is incorrect. The student may have been thinking of a square, for which a 90-degree rotation works, overlooking the fact that even though the diagonals of the rhombus are perpendicular, their lengths are not equal, and therefore, vertexes cannot coincide after the 90° rotation around the center of the rhombus.

Rationale for Second Option: **Key** – The student correctly recognized that since the pairs of opposite vertices are equidistant from the center of the rhombus, a 180-degree rotation will map opposite vertices onto themselves and the entire rhombus onto itself.

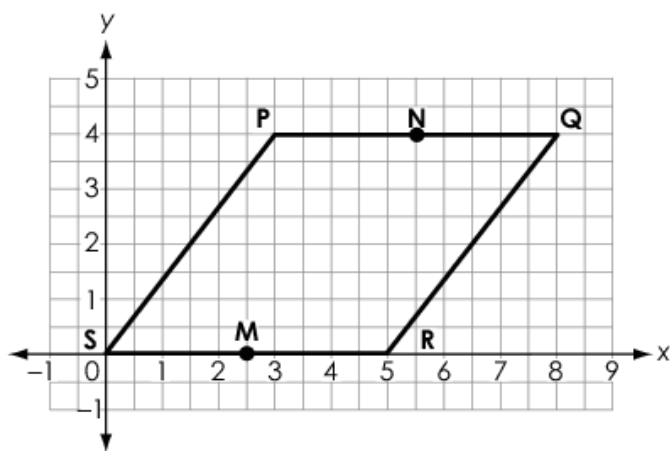
Rationale for Third Option: **Key** – The student correctly recognized that the symmetry in the rhombus allows the reflection across a diagonal to map the figure onto itself, because diagonals of a rhombus are perpendicular and bisect each other.

Rationale for Fourth Option: This is incorrect. The student may have been thinking perhaps of a square or rectangle where the line connecting the midpoints of opposite sides is an axis of symmetry because it would bisect the sides and be perpendicular to the sides. However, this is a rhombus that is not a square, so that does not apply.

Rationale for Fifth Option: **Key** – The student correctly recognized that the symmetry in the rhombus allows the reflection across a diagonal to map the figure onto itself, because diagonals of a rhombus are perpendicular and bisect each other.

Sample Response: 1 point

Rhombus PQRS is shown on the coordinate plane. Points M and N are midpoints of their respective sides.



Select all of the transformations that map the rhombus onto itself.

- ☐ a 90° clockwise rotation around the center of the rhombus
- ☒ a 180° clockwise rotation around the center of the rhombus
- ☒ a reflection across $\overline{PR}$
- ☐ a reflection across $\overline{NM}$
- ☒ a reflection across $\overline{QS}$

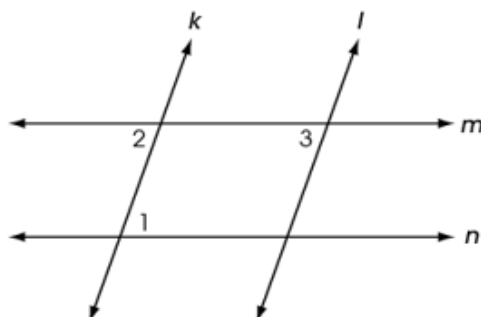
Geometry
Spring 2017 Item Release

Question 13

Question and Scoring Guidelines

Question 13

Two pairs of parallel lines intersect to form a parallelogram as shown.



Place statements and reasons in the table to complete the proof that the opposite angles of a parallelogram are congruent.

Statements		Reasons	
1.	$m \parallel n$ $k \parallel l$	1.	Given
2.		2.	
3.		3.	
4.		4.	

$$\angle 1 \cong \angle 2$$

$$\angle 1 \cong \angle 3$$

$$\angle 2 \cong \angle 3$$

$$\angle 1 \cong \angle 1$$

Alternate exterior angles are congruent.

Alternate interior angles are congruent.

Transitive property of congruence

Opposite angles are congruent.

Corresponding angles are congruent.

Points Possible: 1

Content Cluster: Prove geometric theorems both formally and informally using a variety of methods.

Content Standard: Prove and apply theorems about parallelograms. Theorems include but are not restricted to the following: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals. (G.CO.11)

Scoring Guidelines

Exemplar Response

Statements		Reasons	
1.	$m \parallel n$ $k \parallel l$	1.	Given
2.	$\angle 1 \cong \angle 2$	2.	Alternate interior angles are congruent.
3.	$\angle 2 \cong \angle 3$	3.	Corresponding angles are congruent.
4.	$\angle 1 \cong \angle 3$	4.	Transitive property of congruence

Other Correct Responses

- Line 2 and Line 3 can be switched.
- The sentence "Corresponding angles are congruent." may be added to Reason 4 without penalty, or be acceptable as the only response in Statement 4.

For this item, a full-credit response includes:

- A correct proof (1 point).

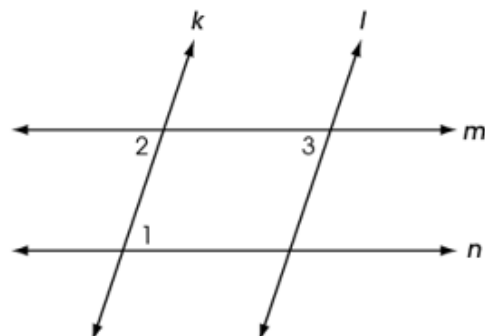
Geometry
Spring 2017 Item Release

Question 13

Sample Responses

Sample Response: 1 point

Two pairs of parallel lines intersect to form a parallelogram as shown.



Place statements and reasons in the table to complete the proof that the opposite angles of a parallelogram are congruent.

Statements		Reasons	
1.	$m \parallel n$ $k \parallel l$	1.	Given
2.	$\angle 1 \cong \angle 2$	2.	Alternate interior angles are congruent.
3.	$\angle 2 \cong \angle 3$	3.	Corresponding angles are congruent.
4.	$\angle 1 \cong \angle 3$	4.	Transitive property of congruence

Alternate exterior angles are congruent.

$$\angle 1 \cong \angle 1$$

Opposite angles are congruent.

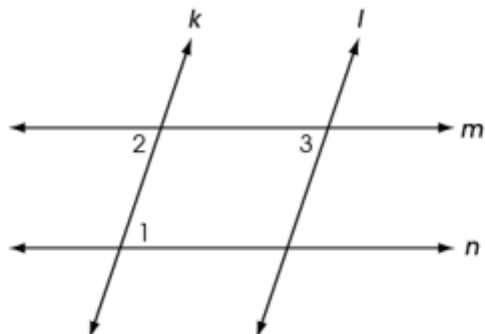
Notes on Scoring

This response earns full credit (1 point) because it correctly completes the proof to show that opposite angles in parallelograms are congruent.

Following from the given information that the pairs of opposite sides in the parallelogram are parallel, there are two pairs of congruent angles formed by parallel lines and a transversal line. For example, if $m \parallel n$, then $\angle 1 \cong \angle 2$ by the Alternate Interior Angles Theorem because the angles are formed by two parallel lines m and n and the transversal line k (step 2). Likewise, if $k \parallel l$, then $\angle 2 \cong \angle 3$ by the Corresponding Angles Theorem because the angles are formed by two parallel lines k and l and the transversal line m (step 3). The proof would also be correct if steps 2 and 3 are switched. Lastly, if $\angle 1 \cong \angle 2$ and $\angle 2 \cong \angle 3$, then $\angle 1 \cong \angle 3$ by the Transitive Property of Congruence (step 4).

Sample Response: 1 point

Two pairs of parallel lines intersect to form a parallelogram as shown.



Place statements and reasons in the table to complete the proof that the opposite angles of a parallelogram are congruent.

Statements		Reasons	
1.	$m \parallel n$ $k \parallel l$	1.	Given
2.	$\angle 2 \cong \angle 3$	2.	Corresponding angles are congruent.
3.	$\angle 1 \cong \angle 2$	3.	Alternate interior angles are congruent.
4.	$\angle 1 \cong \angle 3$	4.	Transitive property of congruence

Alternate exterior angles are congruent.

$$\angle 1 \cong \angle 1$$

Opposite angles are congruent.

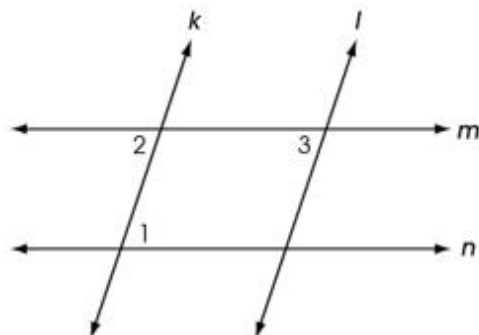
Notes on Scoring

This response earns full credit (1 point) because it correctly completes the proof to show that opposite angles in parallelograms are congruent (with steps 2 and 3 switched).

Following from the given information that the pairs of opposite sides in the parallelogram are parallel, there are two pairs of congruent angles formed by parallel lines and a transversal line. For example, if $k \parallel l$, then $\angle 2 \cong \angle 3$ by the Corresponding Angle Theorem because the angles are formed by two parallel lines k and l and the transversal line m (step 2). Likewise, if $m \parallel n$, then $\angle 1 \cong \angle 2$ by the Alternate Interior Angle Theorem because the angles are formed by two parallel lines m and n and the transversal line k (step 3). Lastly, if $\angle 1 \cong \angle 2$ and $\angle 2 \cong \angle 3$, then $\angle 1 \cong \angle 3$ by the Transitive Property of Congruence (step 4).

Sample Response: 0 points

Two pairs of parallel lines intersect to form a parallelogram as shown.



Place statements and reasons in the table to complete the proof that the opposite angles of a parallelogram are congruent.

Statements		Reasons	
1.	$m \parallel n$ $k \parallel l$	1.	Given
2.	$\angle 2 \cong \angle 3$	2.	Corresponding angles are congruent.
3.	$\angle 1 \cong \angle 2$	3.	Alternate exterior angles are congruent.
4.	$\angle 1 \cong \angle 3$	4.	Transitive property of congruence

Alternate interior angles are congruent.

Opposite angles are congruent.

$$\angle 1 \cong \angle 1$$

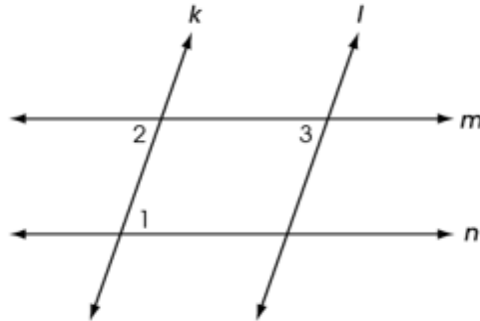
Notes on Scoring

This response earns no credit (0 points) because it shows the incorrect reasoning in step 3 for the proof that opposite angles in parallelograms are congruent.

The student may have confused alternate interior angles with alternate exterior angles.

Sample Response: 0 points

Two pairs of parallel lines intersect to form a parallelogram as shown.



Place statements and reasons in the table to complete the proof that the opposite angles of a parallelogram are congruent.

Statements		Reasons	
1.	$m \parallel n$ $k \parallel l$	1.	Given
2.	$\angle 1 \cong \angle 2$	2.	
3.	$\angle 2 \cong \angle 3$	3.	
4.	$\angle 1 \cong \angle 3$	4.	

$$\angle 1 \cong \angle 1$$

Alternate exterior angles are congruent.

Alternate interior angles are congruent.

Transitive property of congruence

Opposite angles are congruent.

Corresponding angles are congruent.

Notes on Scoring

This response earns no credit (0 points) because it incorrectly completes the proof to show that opposite angles in parallelograms are congruent. Although the proof shows all correct statements, it misses all corresponding reasons for each of the three steps.

Geometry
Spring 2017 Item Release

Question 14

Question and Scoring Guidelines

Question 14

Angle A is the complement of angle B.

Which equation about the two angles must be true?

- Ⓐ $\sin A = \sin B$
- Ⓑ $\sin A = \cos A$
- Ⓒ $\cos B = \sin B$
- Ⓓ $\cos A = \sin B$

Points Possible: 1

Content Cluster: Define trigonometric ratios, and solve problems involving right triangles.

Content Standard: Explain and use the relationship between the sine and cosine of complementary angles. (*G.SRT.7*)

Scoring Guidelines

Rationale for Option A: This is incorrect. The student may have confused complementary angles with congruent angles and concluded that the sine of two congruent angles are equal.

Rationale for Option B: This is incorrect. The student may have recognized that the relationship involves the sine of an angle equaling the cosine of an angle but did not realize that it is the sine of one angle being equal to the cosine of its complement.

Rationale for Option C: This is incorrect. The student may have recognized that the relationship involves the sine of an angle equaling the cosine of an angle but did not realize that it is the sine of one angle being equal to the cosine of its complement.

Rationale for Option D: **Key** – The student correctly noted that the sine of an angle is equal to the cosine of its complement.

Sample Response: 1 point

Angle A is the complement of angle B.

Which equation about the two angles must be true?

- ☐ Ⓐ $\sin A = \sin B$
- ☒ Ⓑ $\sin A = \cos A$
- ☐ Ⓒ $\cos B = \sin B$
- ☐ Ⓓ $\cos A = \sin B$

Geometry
Spring 2017 Item Release

Question 15

Question and Scoring Guidelines

Question 15

Square ABCD has vertices at A (1, 2) and B (3, -3).

What is the slope of $\overline{BC}$?

← → ↶ ↷ ✕

1	2	3
4	5	6
7	8	9
	0	
.	-	$\frac{\Box}{\Box}$

Points Possible: 1

Content Cluster: Use coordinates to prove simple geometric theorems algebraically and to verify specific geometric statements.

Content Standard: Justify the slope criteria for parallel and perpendicular lines, and use them to solve geometric problems, e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point. (G.GPE.5)

Scoring Guidelines

Exemplar Response

- $\frac{2}{5}$

Other Correct Responses

- Any equivalent value

For this item, a full-credit response includes:

- A correct value (1 point).

Geometry
Spring 2017 Item Release

Question 15

Sample Responses

Sample Response: 1 point

Square ABCD has vertices at A (1, 2) and B (3, -3).

What is the slope of $\overline{BC}$?

$\frac{2}{5}$

← → ↶ ↷ ✕

1	2	3
4	5	6
7	8	9
	0	
.	-	$\frac{\Box}{\Box}$

Notes on Scoring

This response earns full credit (1 point) because it shows the correct slope for the line segment perpendicular to the given line segment whose end points' coordinates are also given.

Because of the fact that all angles in a square are 90 degrees, all pairs of adjacent sides are perpendicular to each other and their slopes are opposite reciprocals, except for a pair of horizontal and vertical sides. In this situation, the slope of $\overline{BC}$ is an opposite reciprocal of the slope of $\overline{AB}$. The slope of $\overline{AB}$ can be found by evaluating the slope formula $m = \frac{(y_2 - y_1)}{(x_2 - x_1)}$ for $y_2 = -3$, $y_1 = 2$, $x_2 = 3$, and $x_1 = 1$, or

$$m = \frac{(-3-2)}{(3-1)}$$

$$= -\frac{5}{2}.$$

Therefore, the slope of $\overline{BC}$ is $\frac{2}{5}$.

Sample Response: 1 point

Square ABCD has vertices at A (1, 2) and B (3, -3).

What is the slope of $\overline{BC}$?

0.4

← → ↶ ↷ ✕

1	2	3
4	5	6
7	8	9
	0	
.	-	$\frac{\Box}{\Box}$

Notes on Scoring

This response earns full credit (1 point) because it shows the correct slope for the line segment perpendicular to the given line segment whose end points' coordinates are also given.

Because of the fact that all angles in a square are 90 degrees, all pairs of adjacent sides are perpendicular to each other and their slopes are opposite reciprocals, except for a pair of horizontal and vertical sides. In this situation, the slope of $\overline{BC}$ is an opposite reciprocal of the slope $\overline{AB}$. The slope of $\overline{AB}$ can be found by evaluating the slope formula

$$m = \frac{(y_2 - y_1)}{(x_2 - x_1)} \text{ for } y_2 = -3, y_1 = 2, x_2 = 3, \text{ and } x_1 = 1, \text{ or}$$

$$\begin{aligned} m &= \frac{(-3-2)}{(3-1)} \\ &= -\frac{5}{2} \end{aligned}$$

Therefore, the slope of $\overline{BC}$ is $\frac{2}{5}$ or 0.4.

Sample Response: 0 points

Square ABCD has vertices at A (1, 2) and B (3, -3).

What is the slope of $\overline{BC}$?

$-\frac{2}{5}$

← → ↶ ↷ ✕

1	2	3
4	5	6
7	8	9
	0	
.	-	$\frac{\Box}{\Box}$

Notes on Scoring

This response earns no credit (0 points) because it shows an incorrect slope for the line segment perpendicular to the given one. The student may have used the reciprocal of $-\frac{5}{2}$ but did not realize that the value of the reciprocal has to be the opposite value, or positive $\frac{2}{5}$.

Sample Response: 0 points

Square ABCD has vertices at A (1, 2) and B (3, -3).

What is the slope of $\overline{BC}$?

$-\frac{5}{2}$

← → ↶ ↷ ✕

1	2	3
4	5	6
7	8	9
	0	
.	-	$\frac{\Box}{\Box}$

Notes on Scoring

This response earns no credit (0 points) because it shows an incorrect slope for the line segment perpendicular to the given one. The student may have thought that the slopes of sides $\overline{AB}$ and $\overline{BC}$ are equal instead of being opposite reciprocals and used the slope of the original line segment $\overline{AB}$.

Geometry
Spring 2017 Item Release

Question 16

Question and Scoring Guidelines

Question 16

Kevin asked Olivia what parallel lines are. Olivia responded, “They are lines that never intersect.”

What important piece of information is missing from Olivia’s response?

- Ⓐ The lines must be straight.
- Ⓑ The lines must be coplanar.
- Ⓒ The lines can be noncoplanar.
- Ⓓ The lines form four right angles.

Points Possible: 1

Content Cluster: Experiment with transformations in the plane.

Content Standard: Know precise definitions of ray, angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and arc length. (G.CO.1)

Scoring Guidelines

Rationale for Option A: This is incorrect. The student may have thought it was necessary to define the lines as being straight, not remembering that all lines are always straight.

Rationale for Option B: **Key** – The student correctly noted that the two lines must also be in the same plane (coplanar) in order to be considered parallel lines. If the lines are in different planes, they could still never intersect, but not be considered parallel (skew lines).

Rationale for Option C: This is incorrect. The student may have thought that lines in two different planes that never intersect are considered parallel simply because they never intersect. However, the student did not think about skew lines, which are lines in two different planes that never intersect and are not parallel.

Rationale for Option D: This is incorrect. The student may have confused the definition of parallel lines with perpendicular lines, which would form four right angles when they intersect.

Sample Response: 1 point

Kevin asked Olivia what parallel lines are. Olivia responded, “They are lines that never intersect.”

What important piece of information is missing from Olivia’s response?

- ☐ Ⓐ The lines must be straight.
- ☒ Ⓑ The lines must be coplanar.
- ☐ Ⓒ The lines can be noncoplanar.
- ☐ Ⓓ The lines form four right angles.

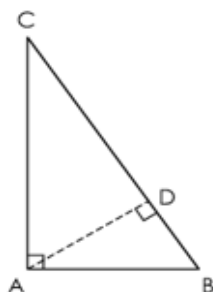
Geometry
Spring 2017 Item Release

Question 17

Question and Scoring Guidelines

Question 17

James correctly proves the similarity of triangles DAC and DBA as shown.



His incomplete proof is shown.

Statements		Reasons	
1.	$m\angle CAB = m\angle ADB = 90^\circ$	1.	Given
2.	$m\angle ADB + m\angle ADC = 180^\circ$	2.	Angles in a linear pair are supplementary.
3.	$90^\circ + m\angle ADC = 180^\circ$	3.	Substitution
4.	$m\angle ADC = 90^\circ$	4.	Subtraction property of equality
5.	$\angle CAB \cong \angle ADB$ $\angle CAB \cong \angle ADC$	5.	Definition of congruent angles
6.	$\angle ABC \cong \angle DBA$ $\angle DCA \cong \angle ACB$	6.	Reflexive property of congruence
7.	$\triangle ABC \sim \triangle DBA$ $\triangle ABC \sim \triangle DAC$	7.	?
8.	$\triangle DBA \sim \triangle DAC$	8.	Substitution

What is the missing reason for the seventh statement?

- (A) CPCTC
- (B) AA postulate
- (C) All right triangles are similar.
- (D) Transitive property of similarity

Points Possible: 1

Content Cluster: Prove and apply theorems both formally and informally involving similarity using a variety of methods.

Content Standard: Prove and apply theorems about triangles. *Theorems include but are not restricted to the following: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity. (G.SRT.4)*

Scoring Guidelines

Rationale for Option A: This is incorrect. The student may have not recognized that CPCTC is the reasoning that follows from congruence, not similarity, and that it has not been proven that the two triangles are congruent.

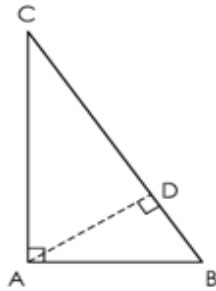
Rationale for Option B: **Key** – The student correctly noticed that James' proof shows that two pairs of corresponding angles are congruent and that this satisfies the AA criterion in showing that two triangles are similar.

Rationale for Option C: This is incorrect. The student may have seen that James has proven that the two right triangles in the graphic are similar, but incorrectly made the generalization that since these two right triangles are similar, all right triangles are similar.

Rationale for Option D: This is incorrect. The student may have chosen the transitive property because the proof shows two similarity statements and one more similarity statement follows after that.

Sample Response: 1 point

James correctly proves the similarity of triangles DAC and DBA as shown.



His incomplete proof is shown.

Statements		Reasons	
1.	$m\angle CAB = m\angle ADB = 90^\circ$	1.	Given
2.	$m\angle ADB + m\angle ADC = 180^\circ$	2.	Angles in a linear pair are supplementary.
3.	$90^\circ + m\angle ADC = 180^\circ$	3.	Substitution
4.	$m\angle ADC = 90^\circ$	4.	Subtraction property of equality
5.	$\angle CAB \cong \angle ADB$ $\angle CAB \cong \angle ADC$	5.	Definition of congruent angles
6.	$\angle ABC \cong \angle DBA$ $\angle DCA \cong \angle ACB$	6.	Reflexive property of congruence
7.	$\triangle ABC \sim \triangle DBA$ $\triangle ABC \sim \triangle DAC$	7.	?
8.	$\triangle DBA \sim \triangle DAC$	8.	Substitution

What is the missing reason for the seventh statement?

- ☐ Ⓐ CPCTC
- ☒ Ⓑ AA postulate
- ☐ Ⓒ All right triangles are similar.
- ☐ Ⓓ Transitive property of similarity

Geometry
Spring 2017 Item Release

Question 18

Question and Scoring Guidelines

Question 18

Triangle ABC is reflected across the line $y = 2x$ to form triangle RST.

Select all of the true statements.

- ☐ $\overline{AB} = \overline{RS}$
- ☐ $\overline{AB} = 2 \bullet \overline{RS}$
- ☐ $\triangle ABC \sim \triangle RST$
- ☐ $\triangle ABC \cong \triangle RST$
- ☐ $m \angle BAC = m \angle SRT$
- ☐ $m \angle BAC = 2 \bullet m \angle SRT$

Points Possible: 1

Content Strand: Understand congruence in terms of rigid motions.

Content Standard: Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent. (G.CO.7)

Scoring Guidelines

Rationale for First Option: **Key** – The student recognized that a reflection is a rigid motion, so the corresponding parts in the two triangles are congruent, and identified a pair of corresponding sides.

Rationale for Second Option: This is incorrect. The student may have thought that reflection is not always a rigid motion and when a figure is reflected across a line $y=2x$, a dilation by a scale factor 2 should be applied to the sides.

Rationale for Third Option: **Key** – The student correctly identified that since a reflection is a rigid motion, the corresponding parts in the two triangles are congruent and since the triangles would be congruent, they would also be similar with scale factor 1.

Rationale for Fourth Option: **Key** – The student recognized that since a reflection is a rigid motion, the corresponding parts in the two triangles are congruent and therefore, the triangles are congruent. Since all of the parts of the two triangles are congruent, the two triangles themselves are congruent.

Rationale for Fifth Option: **Key** – The student recognized that a reflection is a rigid motion, so the corresponding parts in the two triangles are congruent, therefore he or she identified a pair of corresponding angles.

Rationale for Sixth Option: This is incorrect. The student may have thought that reflection is not always a rigid motion and when a figure is reflected across a line $y=2x$, a dilation by a scale factor 2 should be applied to the angle measures.

Sample Response: 1 point

Triangle ABC is reflected across the line $y = 2x$ to form triangle RST.

Select all of the true statements.

- ☒ $\overline{AB} = \overline{RS}$
- ☐ $\overline{AB} = 2 \cdot \overline{RS}$
- ☒ $\triangle ABC \sim \triangle RST$
- ☒ $\triangle ABC \cong \triangle RST$
- ☒ $m\angle BAC = m\angle SRT$
- ☐ $m\angle BAC = 2 \cdot m\angle SRT$

Geometry
Spring 2017 Item Release

Question 19

Question and Scoring Guidelines

Question 19

The equation of a circle is shown.

$$x^2 + y^2 - 10x + 8y + 16 = 0$$

What is the radius of the circle?

radius:

The interface includes a toolbar with navigation and editing icons (undo, redo, left arrow, right arrow, and delete). Below the toolbar is a keypad with the following buttons:

1	2	3	+	-	•	÷					
4	5	6	<	≤	=	≥	>				
7	8	9	$\frac{\Box}{\Box}$	$\Box^\Box$	$\Box_\Box$	()		$\sqrt{\Box}$	$\sqrt[\Box]{\Box}$	π	i
0	.	-	sin	cos	tan	arcsin	arccos	arctan			

Points Possible: 1

Content Cluster: Translate between the geometric description and the equation for a conic section.

Content Standard: Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.
(G.GPE.1)

Scoring Guidelines

Exemplar Response

- 5

Other Correct Responses

- Any equivalent value

For this item, a full-credit response includes:

- A correct value (1 point).

Geometry
Spring 2017 Item Release

Question 19

Sample Responses

Sample Response: 1 point

The equation of a circle is shown.

$$x^2 + y^2 - 10x + 8y + 16 = 0$$

What is the radius of the circle?

radius:

5

← → ↶ ↷ ✕											
1	2	3	+	-	•	÷					
4	5	6	<	≤	=	≥	>				
7	8	9	$\frac{\Box}{\Box}$	$\Box^\Box$	$\Box_\Box$	()		$\sqrt{\Box}$	$\sqrt[\Box]{\Box}$	π	i
0	.	-	sin	cos	tan	arcsin	arccos	arctan			

Notes on Scoring

This response earns full credit (1 point) because it shows the correct radius of the circle.

This question can be answered by converting the given equation to the center-radius form $(x - h)^2 + (y - k)^2 = r^2$, where (h, k) is the center and r is the radius of the circle. The conversion to the center-radius form employs the process of completing the square relative to the variable x and to the variable y . The first step is to subtract 16 from both sides of the equation to get $x^2 + y^2 - 10x + 8y = -16$. The second step is to group terms $(x^2 - 10x) + (y^2 + 8y) = -16$. The third step is to add the extra values to each set of parentheses in order to complete the square. The calculations for the two extra values are as follows: $(\frac{-10}{2})^2 = 25$ and $(\frac{8}{2})^2 = 16$, so 25 and 16 need to be added to the left side of the equation. The same values also must be added to the right side to produce an equivalent equation.

Now, the equation looks like
 $(x^2 - 10x + 25) + (y^2 + 8y + 16) = -16 + 25 + 16$.

By factoring two trinomials individually as $x^2 - 10x + 25 = (x - 5)^2$ and $y^2 + 8y + 16 = (y + 4)^2$, the process of completing the square is finished and the given equation is in center-radius form $(x - 5)^2 + (y - 4)^2 = 5^2$. From here, the radius of the circle is 5.

Sample Response: 1 point

The equation of a circle is shown.

$$x^2 + y^2 - 10x + 8y + 16 = 0$$

What is the radius of the circle?

radius:

← → ↶ ↷ ✕											
1	2	3	+	-	•	÷					
4	5	6	<	≤	=	≥	>				
7	8	9	$\frac{\Box}{\Box}$	$\Box^\Box$	$\Box_\Box$	()		$\sqrt{\Box}$	$\sqrt[\Box]{\Box}$	π	i
0	.	-	sin	cos	tan	arcsin	arccos	arctan			

Notes on Scoring

This response earns full credit (1 point) because it shows the correct radius of the circle.

This question can be answered by converting the given equation to the center-radius form $(x - h)^2 + (y - k)^2 = r^2$, where (h, k) is the center and r is the radius of the circle. The conversion to the center-radius form employs the process of completing the square relative to the variable x and to the variable y . The first step is to subtract 16 from both sides of the equation to get $x^2 + y^2 - 10x + 8y = -16$. The second step is to group terms $(x^2 - 10x) + (y^2 + 8y) = -16$. The third step is to add the extra values to each set of parentheses in order to complete the square. The calculations for the two extra values are as follows: $(\frac{-10}{2})^2 = 25$ and $(\frac{8}{2})^2 = 16$, so 25 and 16 need to be added to the left side of the equation. The same values also must be added to the right side to produce an equivalent equation.

Now, the equation looks like

$$(x^2 - 10x + 25) + (y^2 + 8y + 16) = -16 + 25 + 16 .$$

By factoring two trinomials individually as $x^2 - 10x + 25 = (x - 5)^2$ and $y^2 + 8y + 16 = (y + 4)^2$ the process of completing the square is finished and the given equation is in center-radius form $(x - 5)^2 + (y - 4)^2 = 5^2$. From here, the radius of the circle is $\sqrt{25}$ or 5.

Sample Response: 0 points

The equation of a circle is shown.

$$x^2 + y^2 - 10x + 8y + 16 = 0$$

What is the radius of the circle?

radius: 4

← → ↶ ↷ ✕									
1	2	3	+	-	•	÷			
4	5	6	<	≤	=	≥	>		
7	8	9	$\frac{\Box}{\Box}$	$\Box^\Box$	$\Box_\Box$	()		$\sqrt{\Box}$	$\sqrt[\Box]{\Box}$
0	.	-	sin	cos	tan	arcsin	arccos	arctan	

Notes on Scoring

This response earns no credit (0 points) because it shows an incorrect radius of the circle. The student may have taken the square root of 16, forgetting to rewrite the equation in the center-radius form before taking the square root of 16.

Sample Response: 0 points

The equation of a circle is shown.

$$x^2 + y^2 - 10x + 8y + 16 = 0$$

What is the radius of the circle?

radius: 25

The interface includes a toolbar with five icons: left arrow, right arrow, undo, redo, and a delete icon (X). Below the toolbar is a grid of buttons for mathematical input:

1	2	3	+	-	•	÷					
4	5	6	<	≤	=	≥	>				
7	8	9	$\frac{\Box}{\Box}$	$\Box^\Box$	$\Box_\Box$	()		$\sqrt{\Box}$	$\sqrt[\Box]{\Box}$	π	i
0	.	-	sin	cos	tan	arcsin	arccos	arctan			

Notes on Scoring

This response earns no credit (0 points) because it shows an incorrect radius of the circle. The student may not have taken the square root of 25.

Geometry
Spring 2017 Item Release

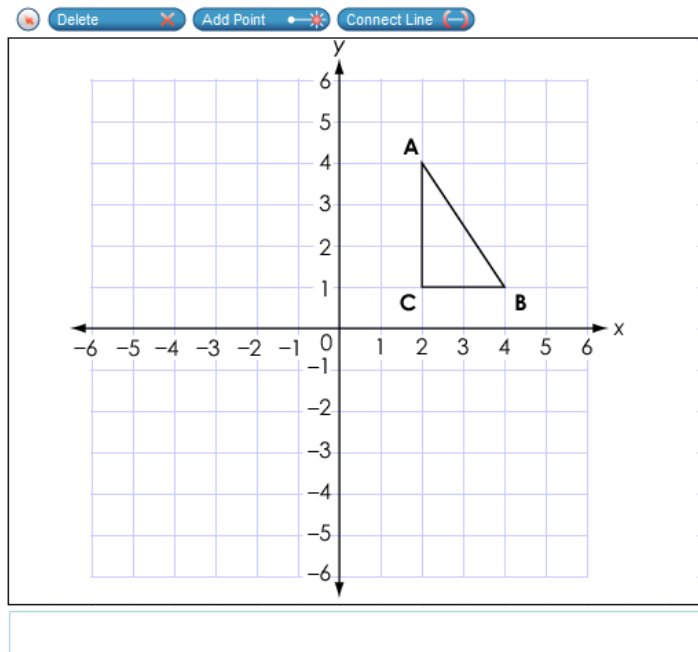
Question 20

Question and Scoring Guidelines

Question 20

Triangle ABC is reflected across the line $y = x$.

Use the Connect Line tool to create the resulting triangle on the coordinate grid.



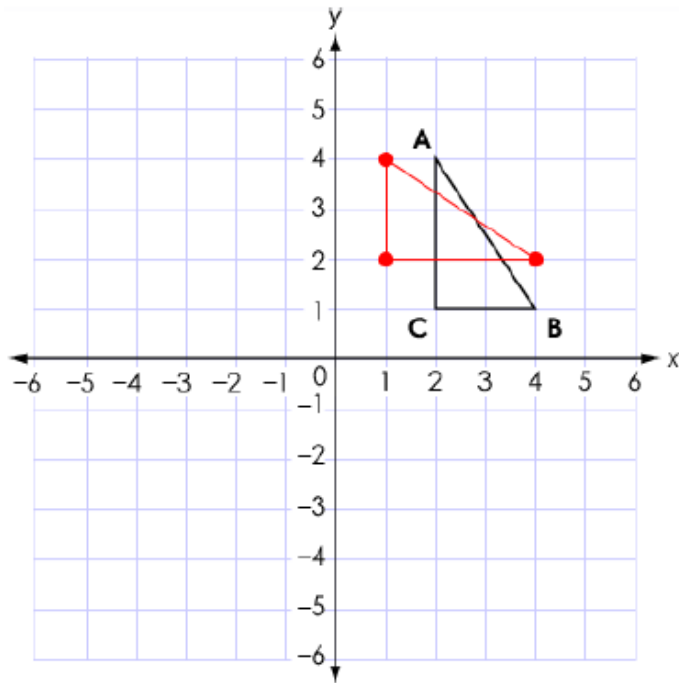
Points Possible: 1

Content Cluster: Experiment with transformation in the plane.

Content Standard: Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using items such as graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another. (G.CO.5)

Scoring Guidelines

Exemplar Response



Other Correct Responses

- N/A

For this item, a full-credit response includes:

- The correct triangle (1 point).

Geometry
Spring 2017 Item Release

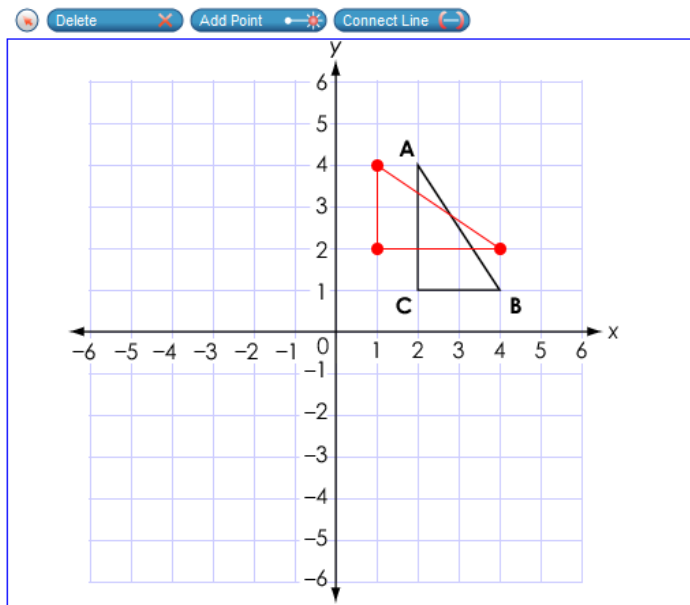
Question 20

Sample Responses

Sample Response: 1 point

Triangle ABC is reflected across the line $y = x$.

Use the Connect Line tool to create the resulting triangle on the coordinate grid.



Notes on Scoring

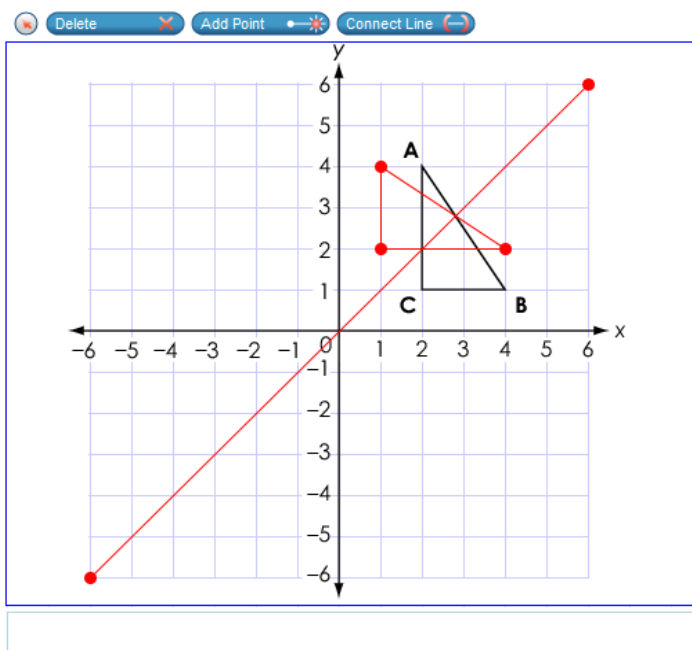
This response earns full credit (1 point) because it shows the correct image of triangle ABC reflected across the line $y = x$. The construction of the resulting triangle is based on the previous knowledge that under a reflection across the line, a point and its image are always the same perpendicular distance from the reflection line. In this situation, after the reflection across the line $y = x$, each vertex (x, y) of the triangle ABC corresponds with the vertex (y, x) of its image, or the x and the y coordinates swap over.

Following from this, the image of A(2, 4) is the point (4, 2), the image of B(4, 1) is the point (1, 4) and the image of C(2, 1) is the point (1, 2). By connecting the three image points, the student creates the resulting triangle. Also, by drawing the auxiliary line $y = x$, the student can observe that each vertex and its image is the same perpendicular distance from the reflection line $y = x$.

Sample Response: 1 point

Triangle ABC is reflected across the line $y = x$.

Use the Connect Line tool to create the resulting triangle on the coordinate grid.



Notes on Scoring

This response earns full credit (1 point) because it shows the correct image of triangle ABC reflected across the line $y = x$ and the line of reflection. The construction of the resulting triangle is based on the previous knowledge that under a reflection across the line, a point and its image are always the same perpendicular distance from the reflection line. In this situation, after the reflection across the line $y = x$, each vertex (x, y) of the triangle ABC corresponds with the vertex (y, x) of its image, or the x and the y coordinates swap over.

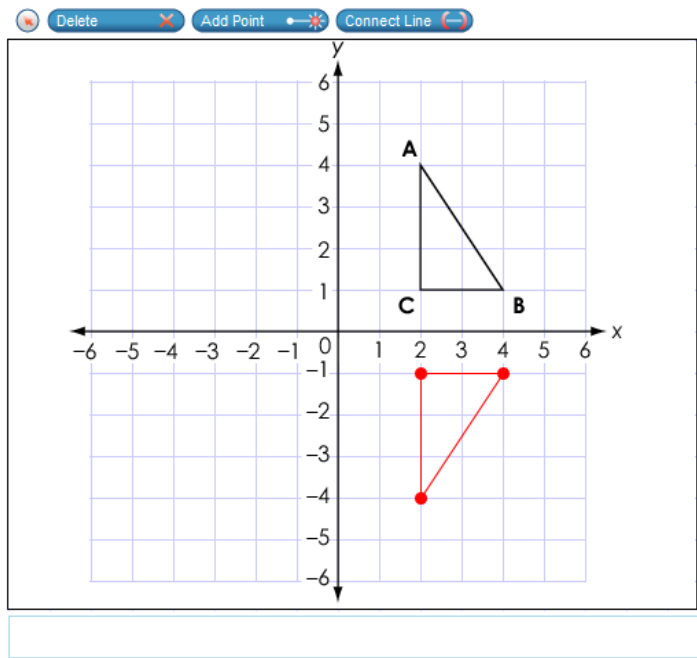
Following from this, the image of A(2, 4) is the point (4, 2), the image of B(4, 1) is the point (1, 4) and the image of C(2, 1) is the point (1, 2). By connecting the three image points, the student creates the resulting triangle. Also, by drawing the auxiliary line $y = x$, the student can observe that each vertex and its image is the same perpendicular distance from the reflection line $y = x$.

A presence of the line $y = x$ on the coordinate grid does not impact the scoring of the item and is not required.

Sample Response: 0 points

Triangle ABC is reflected across the line $y = x$.

Use the Connect Line tool to create the resulting triangle on the coordinate grid.



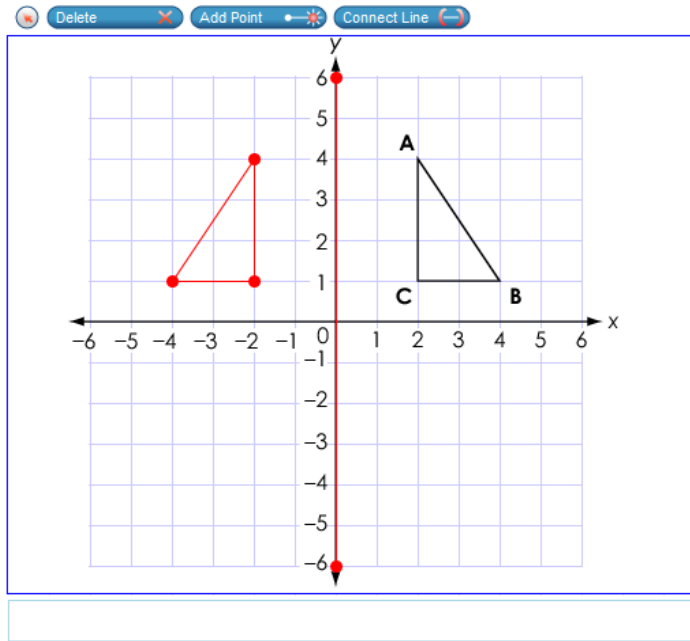
Notes on Scoring

This response earns no credit (0 points) because it shows an incorrect image of triangle ABC. The response shows the image of triangle ABC reflected across the x-axis instead of it reflected across the line $y = x$.

Sample Response: 0 points

Triangle ABC is reflected across the line $y = x$.

Use the Connect Line tool to create the resulting triangle on the coordinate grid.



Notes on Scoring

This response earns no credit (0 points) because it shows an incorrect image of triangle ABC. The response shows the image of triangle ABC reflected across the y-axis instead of it reflected across the line $y = x$.

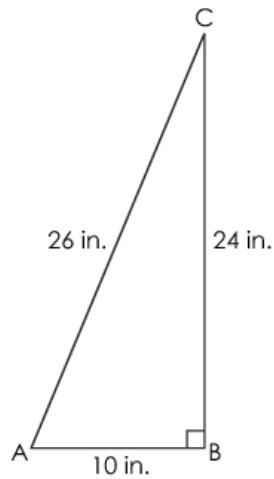
Geometry
Spring 2017 Item Release

Question 21

Question and Scoring Guidelines

Question 21

Triangle ABC is shown.



What is $\tan(A)$?

← → ↶ ↷ ✕

1	2	3
4	5	6
7	8	9
	0	
.	-	$\frac{\Box}{\Box}$

Points Possible: 1

Content Cluster: Define trigonometric ratios, and solve problems involving right triangles.

Content Standard: Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles. (G.SRT.6)

Scoring Guidelines

Exemplar Response

- 2.4

Other Correct Responses

- Any equivalent value

For this item, a full-credit response includes:

- A correct value (1 point).

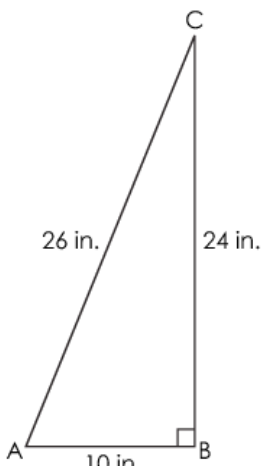
Geometry
Spring 2017 Item Release

Question 21

Sample Responses

Sample Response: 1 point

Triangle ABC is shown.



What is $\tan(A)$?

2.4

← → ↶ ↷ ✕

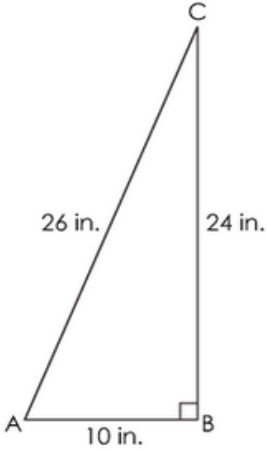
1	2	3
4	5	6
7	8	9
	0	
.	-	$\frac{\Box}{\Box}$

Notes on Scoring

This response earns full credit (1 point) because it shows the correct $\tan(A)$. In the right triangles, the tangent of the acute angle is the ratio of the lengths of the opposite leg to the adjacent leg. Based on this definition, $\tan(A) = \frac{CB}{AB} = \frac{24}{10}$ or $\frac{12}{5}$ or 2.4.

Sample Response: 1 point

Triangle ABC is shown.



What is $\tan(A)$?

$\frac{24}{10}$

← → ↶ ↷ ✕

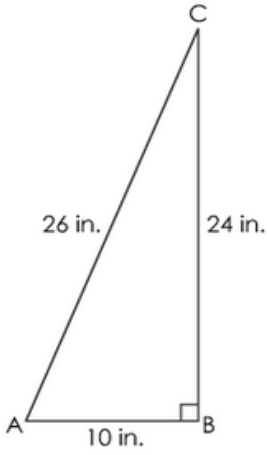
1	2	3
4	5	6
7	8	9
	0	
.	-	$\frac{\Box}{\Box}$

Notes on Scoring

This response earns full credit (1 point) because it shows the correct $\tan(A)$. In the right triangles, the tangent of the acute angle is the ratio of the lengths of the opposite leg to the adjacent leg. Based on this definition, $\tan(A) = \frac{CB}{AB} = \frac{24}{10}$.

Sample Response: 0 points

Triangle ABC is shown.



What is $\tan(A)$?

$\frac{10}{24}$

← → ↶ ↷ ✕

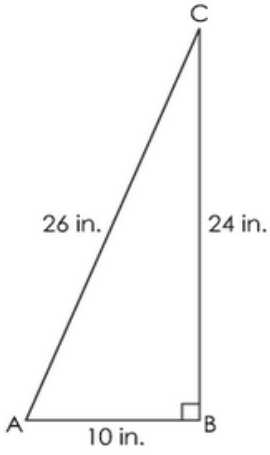
1	2	3
4	5	6
7	8	9
	0	
.	-	$\frac{\Box}{\Box}$

Notes on Scoring

This response earns no credit (0 points) because it shows an incorrect $\tan(A)$. In the right triangles, the tangent of the acute angle is the ratio of the lengths of the opposite leg to the adjacent leg. In this situation, the student may have found the reciprocal of the tangent, or the ratio of the lengths of the adjacent leg to the opposite leg.

Sample Response: 0 points

Triangle ABC is shown.



What is $\tan(A)$?

$\frac{24}{26}$

← → ↶ ↷ ✕

1	2	3
4	5	6
7	8	9
	0	
.	-	$\frac{\Box}{\Box}$

Notes on Scoring

This response earns no credit (0 points) because it shows an incorrect $\tan(A)$. In the right triangles, the tangent of the acute angle is the ratio of the lengths of the opposite leg to the adjacent leg. In this situation, the student may have found the $\sin(A)$, or the ratio of the lengths of the opposite leg to the hypotenuse.

The Ohio Department of Education does not discriminate on the basis of race, color, national origin, sex, religion, age, or disability in employment or the provision of services.

Copyright © 2017 by the Ohio Department of Education. All rights reserved.